Estimation of Currency Swap Yield Curve

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Abstract

Currency swap is a financial derivative widely utilized to manage foreign exchange liquidity and to conduct carry trade transactions. Besides, central banks and investors follow currency swap market for the purposes of valuing financial derivatives, estimating counterparty risk and inferring about monetary policy stance. Therefore, it is crucial to interpret the information related to currency swap rates. However, currency swap rates are quoted as par-rate, and their interpretation is not straightforward. This study employs one of the most popular parametric yield curve estimation methods, Nelson-Siegel model, for currency swap rates to form a zero-coupon currency swap yield curve. The results show the fitted and quoted currency swap rates are quite close to each other. Additionally, the zero-coupon swap rates are compared with forward implied rates for specific maturities since both products are quite similar in nature. Both rates are observed to move together, which shows the consistency of our estimations.

Keywords: Currency swap, Yield Curve, Nelson-Siegel

Öz

Kur takası, özellikle döviz likiditesini yönetme ve taşıma ticareti (carry trade) işlemlerini yapabilme amaçlarıyla oldukça yaygın bir şekilde kullanılan bir türev enstrümandır. Ayrıca, merkez bankaları ve yatırımcılar da kur takası piyasasını türev ürünleri değerlemek, karşı taraf riski hesaplamak ve para politikası duruşu hakkında bilgi çıkarmak amaçlarıyla yakından takip etmektedirler. Bu sebeple, kur takası faiz oranlarındaki değişimleri doğru yorumlamak oldukça önemlidir. Ancak, kur takası oranları par-faiz oranları olduğu için doğrudan yorumlamak çok kolay değildir. Bu çalışmada, getiri eğrisi tahmininde oldukça yaygın olarak kullanılan parametrik Nelson-Siegel modeli ile spot kur takası getiri eğrisi tahmin edilmektedir. Sonuçlar kote edilen ve tahmin edilen oranların birbirlerine oldukça yakın gerçekleştiklerini göstermektedir. Ek olarak, spot kur takası faiz oranları ile kur forwardların ima ettiği faiz oranları doğası gereği oldukça benzer ürünler olmalarından dolayı belirli vadeler için karşılaştırılmıştır. Bulgular belirtilen faiz oranlarının birbirlerine oldukça yakın olduğunu göstermektedir.

Anahtar Kelimeler: Kur takası, Getiri eğrisi, Nelson-Siegel

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I. Introduction

Currency swap is a financial derivative contract that enables the parties to transform assets or liabilities in one currency into another one. This product has been extensively used in global markets for the main purposes of managing foreign exchange liquidity and conducting carry trade transactions. According to BIS Triennial Survey, as of 2016 foreign exchange swaps constitute almost half of the total foreign currency instruments (Table 1).

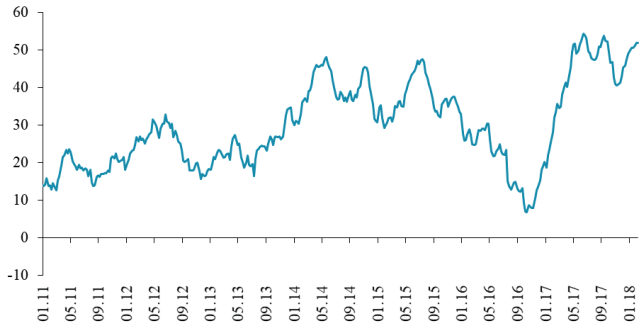
Table 1. OTC Foreign Exchange Turnover
(Daily averages in April, Billion USD)

Instrument	2001	2004	2007	2010	2013	2016
Foreign Currency Instruments	1,239	1,934	3,324	3,973	5,357	5,067
Spot transactions	386	631	1,005	1,489	2,047	1,652
Outright forwards	130	209	362	475	679	700
Foreign Exchange Swaps	**656**	**954**	**1,714**	**1,759**	**2,240**	**2,378**
Currency Swaps	**7**	**21**	**31**	**43**	**54**	**82**
Options and other products	60	119	212	207	337	254

Source: BIS Triennial Survey

This contract has a widespread use by the banks and real sector firms in Turkey as well. The banks and firms generally conduct currency swap contracts for liquidity management. The counterparties in these transactions happen to be non-residents who generally engage in currency swap contracts to invest in Turkish Lira denominated assets. As of February 2018, the size of these transactions totals more than 50 billion USD (Figure 1).

Figure 1. Off-Balance Sheet Position of Turkish Banking System[4]
(Billion USD)



Source: BRSA. Last Observation: February 2018

[4] The off-balance sheet position of Turkish banks is a proxy for currency swap transactions.

Given the high volume of these transactions, it is crucial to understand the dynamics of the currency swap contracts. The value of these contracts depends mainly on the movements along domestic and foreign yield curves and the changes in exchange rate. Additionally, the central banks and other financial agents use the information in currency swap rates to extract expectations about monetary policy stance. Therefore, forming a yield curve for currency swap rates is crucial. However, interpreting the movements on the quoted currency swap rates is not straightforward. The currency swap rates are par-rates, which are affected by the movements on the yield curve up to the maturity. Therefore, it is important to extract the zero coupon currency swap rates to interpret them easily. Since there is no sufficient amount of observation along the maturity spectrum in the currency swap market, we rely on parametric yield curve estimation methodologies to form a continuous zero coupon curve. In this regard, this study provides an approach to form yield curve for currency swaps using one of the most popular parametric yield curve methodologies, Nelson-Siegel model.

Most of the previous studies focus on estimating yield curve for Treasury bond market rather than the currency swap markets. In those studies, it is found out that the Nelson-Siegel type of parametric models are successful at capturing the movements along the yield curve (Nelson and Siegel, 1987). Additionally, BIS study (2005) shows that many central banks employ Nelson-Siegel or its extended version to estimate yield curve. In Turkey, Akinci et al. (2006) provide an application of Nelson-Siegel yield curve methodology for Turkish Treasury bonds using the zero-coupon and fixed coupon bonds. Cepni and Kucuksarac (2017) and Kucuksarac (2017) show that the in-sample and out-of-sample errors of Extended Nelson-Siegel model are reasonably low. Although various efforts have been put on estimation of yield curve from Turkish Treasury bonds, there is no study about the yield curve formation for currency swaps, to the best of our knowledge. This study has a distinct feature as it will be the first one estimating currency swap yield curve for Turkish market.

II. Data and Methodology

Currency swaps are mostly traded on over-the-counter (OTC) markets. Therefore, we rely on the Bloomberg quotations for currency swap rates. The quotations are available at various maturities. In this study, we use daily closing quotes for currency swap rates at maturities of 1, 2, 3, 6, 12, 24, 36, 48, 60, 72, 84, 96 and 120 months[5]. Since the currency swaps for longer maturities are available after January 2011, the sample period consists of observations between January 2011 and December 2017.

[5] Bloomberg quotations are fixed par rates against floating LIBOR.

Cross currency swaps are agreements to exchange interest payments and principals denominated in two different currencies. The interest payments are exchanged at the predetermined dates through the life of the swap and they can be floating or fixed. However, the structure which one leg is fixed and the other one is floating is the most commonly used one. The principal amounts in each currency are usually exchanged at the beginning and at the end of the life of the swap. They are chosen to be equivalent using the exchange rate at the currency swap's initiation.

The prices of currency swaps are usually quoted as the interest rate on the fixed leg against the LIBOR on the floating leg. The frequency of floating payments is quarterly whereas the frequency is annual for fixed payments. However, there are no intermediate cash flows if the maturity of the swap is shorter than six months. Cash flow structure of a currency swap with the notional amount of 1 unit of foreign currency is illustrated in Table 2.

Table 2. Mechanics of a Cross Currency Swap
(Notional Amount of Cross Currency Swap is 1 Unit of Foreign Currency)

	Cash Flows at Deal Date	**Periodic Cash Flows**	**Cash Flow at Maturity**
In Foreign Currency	+1	$-L(t_{i-1}, t_{i)}\tau$	$-1 - L(t_{m-1}, t_m)\tau$
In Domestic Currency	$-S_{t_0}$	$+S_{t_0}.r_{swap,D}\tau$	$S_{t_0}+S_{t_0}.r_{swap,D}\tau$

In this case, $L(t_{i-1}, t_i)$ and $r_{swap,D}$ denote the foreign and domestic currency swap interest rates and S_{t_0} stands for the value of domestic currency for one unit of foreign currency at the initiation of swap contract.

The value of a "receive fixed, pay floating" swap at time t_0 (V_{t_0}) can be replicated by a portfolio consisting of a long position in a fixed rate coupon bond in domestic currency $(V_{t_0,fixed\ leg})$ and a short position in a floating rate coupon bond in foreign currency $(V_{t_0,floating\ leg})$.[6] Thus, the value of the swap initiated at time t_0 can be expressed as the difference between these two bonds.

$$V_{t_0} = V_{t_0,fixed\ leg} - V_{t_0,floating\ leg}\, S_{t_0} \tag{1}$$

$$V_{t_0,fixed\ leg} = \sum_{i=1}^{m} S_{t_0} Nc\tau D(t_0, t_i) + S_{t_0} N\, D(\, t_0, t_m) \tag{2}$$

$$V_{t_0,floating\ leg} = \sum_{i=1}^{m} NL(t_{i-1}, t_i)\, \tau D^f(\, t_0, t_i) + N\, D^f(\, t_0, t_m) \tag{3}$$

[6] "Receive fixed, pay floating" swap stands for the transactions which fixed rate payments in domestic currency are exchanged for floating rate payments in foreign currency.

In this representation, N denotes the notional amount of the foreign currency bonds. $D(t_0, t_i)$ and $D^f(t_0, t_i)$represent the discount functions for the domestic and foreign currency and $L(t_{i-1}, t_i)$ stands for the floating rate for the period between t_{i-1} and t_i The currency swap rate is denoted by c and τ represents the coupon payment period in terms of years.

The floating interest rates are not observable when the swap is initiated. However, the valuation of floating leg can be found out by replicating its cash flows. In this respect, the following portfolio generates the same cash flow with the floating cash flow at time t_i, $NL(t_{i-1}, t_i)\,\tau$.

- At time t_0, invest in the zero-coupon bond with maturity t_{i-1} and invest the proceeds N at time t_{i-1} at the rate of $L(t_{i-1}, t_i)$
- At time t_0, short the zero-coupon bond with maturity t_i

The value of the cash flow, $NL(t_{i-1}, t_i)\tau$, is equal to $N\left\{D^f(t_0, t_{i-1}) - D^f(t_0, t_i)\right\}$at time t_0. Then, the sum of the value of all floating cash flows is equal to

$$V_{t_0, floating\ leg} = \sum_{i=1}^{m} N\left\{D^f(t_0, t_{i-1}) - D^f(t_0, t_i)\right\} + N\,D^f(t_0, t_m) \tag{4}$$

The value of the floating leg becomes equal to its par value at the beginning of the contract and at the coupon dates. Then, the value of a currency swap at time t_0 can be shown as below:

$$V_{t_0} = \sum_{i=1}^{m} S_{t_0} N\tau c D(t_0, t_i) + S_{t_0} N\,D(t_0, t_m) - \left(\sum_{i=1}^{m} S_{t_0} N\left\{D^f(t_0, t_{i-1}) - D^f(t_0, t_i)\right\}\right. - S_{t_0} N D^f(t_0, t_m) \tag{5}$$

At the initiation of the contract, the value of a currency swap (Equation 6) is zero. Therefore the fixed rate on currency swap (Equation 7) is equal to:

$$V_{t_0} = \sum_{i=1}^{m} N\tau c S_{t_0} D(t_0, t_i) + S_{t_0} N\,D(t_0, t_m) - S_{t_0} N = 0 \tag{6}$$

$$c = \frac{1}{\tau}\,\frac{1 - D(t_0, t_m)}{\sum_{i=1}^{m} D(t_0, t_i)} \tag{7}$$

As can be seen from the equation, currency swap rates are par-rates rather than zero-coupon rates. Therefore, currency swap rate is a function of the yield curve up to the maturity of the swap. This makes it difficult to interpret the currency swap rate movements. Hence, it is necessary to find out the zero rates. In this regard, we utilize Nelson-Siegel methodology.

Nelson-Siegel methodology, which has been extensively used for Treasury bond market yield curve construction, relies on a functional form consistent with the characteristics of the interest rates observed in the market (Equation 8). Specifically, the model assumes that the zero rates can be described explicitly by the following functional form:

$$r(m,\beta,\tau) = \beta_0 + \beta_1\left(\frac{1-e^{-\frac{m}{\tau}}}{\frac{-m}{\tau}}\right) + \beta_2\left(\frac{1-e^{\frac{-m}{\tau}}}{\frac{-m}{\tau}} - e^{\frac{-m}{\tau}}\right) \quad (8)$$

where m denotes the time to maturity, β=$(\beta_0,\beta_1,\beta_2,\tau)$ is the parameter set to be estimated. Since the price of the floating rate bond is at par at the initiation of the contract, we can treat the fixed leg of the cross currency swap as the fixed rate bond with coupon rate equal to the currency swap rates. Therefore, the value of the fixed leg of the currency swap will be equal to the par value.

To find out the optimal NS parameter set for zero-coupon swap curve, we minimize the price difference between the actual prices (which is equal to the par value in this application) and fitted price weighted by the inverse of the duration (Equation 9). The duration is equal to the Macaulay duration of the fixed leg of the currency swap, in other words the fixed rate bond with coupon rate equal to the currency swap rate. The objective function is

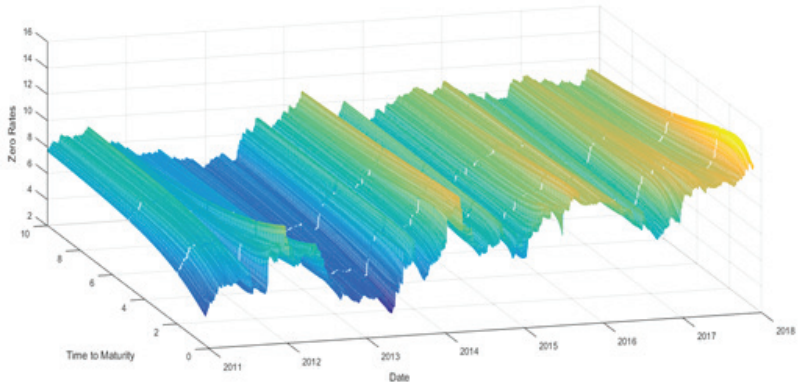
$$min_\beta \ \sum_{i=1}^{K}\left(\frac{P_{t_0}^{i} - P_{t_0}^{i,fitted}}{D_{t_0}^{i}}\right)^2 \quad (9)$$

where D_t^i denotes the Macaulay duration at time t_0, K stands for the number of quoted cross currency swap rates and $P_{t_0}^i$ is equal to par value at initiation of the swap.

III. Empirical Evidence

The main focus point of the study is to extract the zero-coupon swap rates embedded in the cross currency swap quotations. Figure 2 shows the evolution of zero-coupon currency swap rate yield curve from January 2011 to the end of December 2017. When we take a look at the current swap yield curve, it can be observed that it is negatively sloped, which means that the long-end rates are lower than the short-tend rates. Additionally, it is observed that the rates tend to display mean-reverting behavior although mean-reversion frequency tends to change over time.

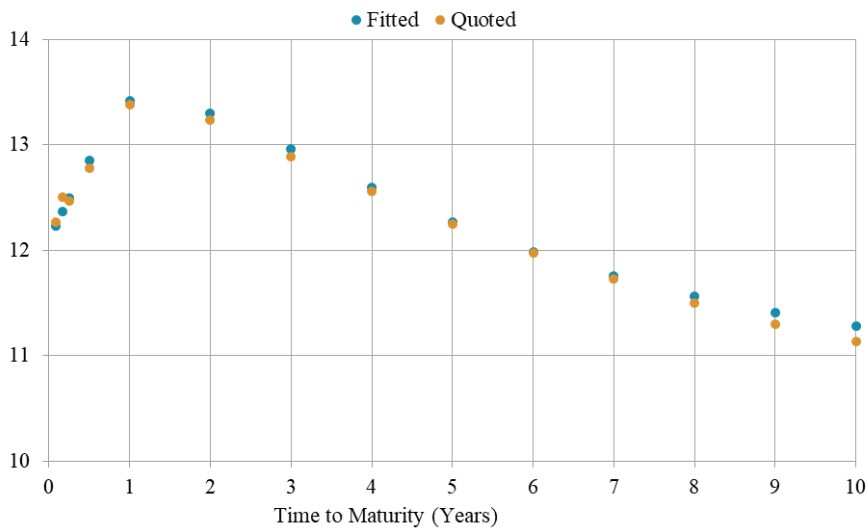
Figure 2. The Zero-Coupon Currency Swap Yield Curve



Source: Authors' Calculations

Figure 3 shows an example of the quoted currency swap rates and fitted par-coupon swap rates. In this respect, we convert the zero-coupon currency swap rates to the par-coupon rates. As can be seen from the graph, the fitted and quoted currency swap rates are quite close to each other although there are some small differences at the very short-end of the curve.

Figure 3. Fitted vs Quoted Swap Rates
(Percent, As of 16 February 2018)



Source: Bloomberg, Authors' Calculations.

Next, we compare the in-sample fit of Nelson Siegel methodology with the quoted currency swap rates. Table 3 shows the differences between the quoted and fitted par rates. As can be seen from the Table, the average errors tend to be quite close to 4 basis points. However, the errors tend to be large around 0-1 years compared to other maturities. The mean absolute errors tend to be around 6 basis points but the errors tend to be smaller as the maturity gets longer. The results indicate that the methodology can be used for the practitioners in order to price similar securities. Additionally, the central banks can use these rates for the purposes of extracting inflation expectations or monetary policy expectations in a reliable manner.

Table 3. Comparison of Errors and Mean Absolute Errors for Different Maturities[7]

	Errors			**Mean Absolute Errors**		
	Average	**Median**	**Standard Errors**	**Average**	**Median**	**Standard Errors**
0-1 Years	0.07	0.07	0.13	0.11	0.09	0.09
1-2 Years	0.00	0.00	0.06	0.05	0.04	0.04
2-5 Years	0.02	0.03	0.04	0.04	0.03	0.03
5-10 Years	0.04	0.03	0.05	0.05	0.04	0.05
Overall	0.04	0.03	0.08	0.06	0.04	0.07

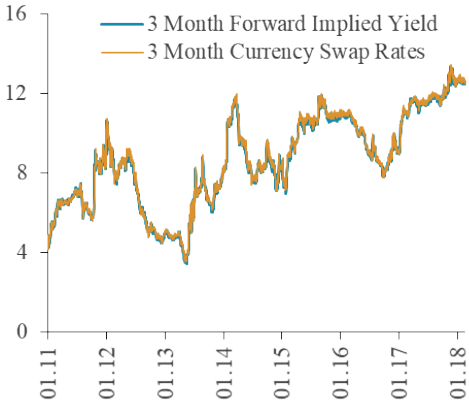
Source: Authors' Calculations

Lastly, we also compare the zero-coupon swap rates with currency forward implied rates for specific maturities[8]. FX forwards are equivalent to zero-coupon swap rates ignoring the liquidity differences and counterparty risk. Since both products are quite similar in nature, they are expected to move together and be closer to each other. In this purpose, we convert the forward implied rates to continuously compounded rates since Nelson-Siegel provides continuously compounded zero rates. The Figure 4 shows that the fitted zero-coupon swap rates and forward implied rates are quite close to each other, which shows the consistency of the yield curve estimation for currency swap rates. Additionally, the results show that the prices in FX forward and currency swap markets do not deviate from no-arbitrage condition, which shows the efficiency in these offshore markets.

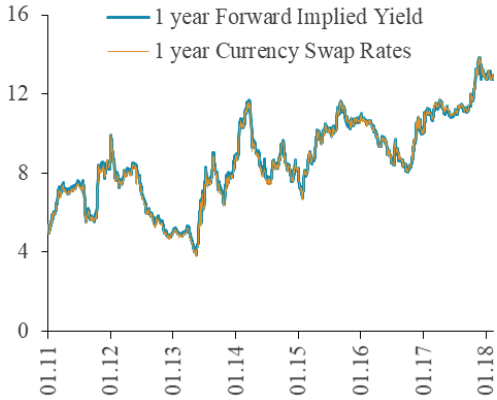
[7] The difference between fitted and quoted swap rates are not statistically significant from zero.

[8] FX forward implied rate indicates the embedded domestic currency rates in FX forward transactions.

Figure 4. Comparison of Forward Implied Yields and Currency Swap Rates
(Continuously Compounded Rates, Percent)



Source: Bloomberg, Authors' Calculations.



Source: Bloomberg, Authors' Calculations.

IV. Conclusion

Regarding the substantial amount of position holding of currency swaps in Turkey, understanding the movements on the swap yield curve is crucial for investors and regulatory authorities. However interpreting the currency swap rates is not straightforward since they are quoted as par-rate, which is a function of the yield curve up to specific maturity, rather than zero rates. This study aims to provide zero-coupon currency swap rates, which are much more easily interpretable. In this regard, we employ Nelson Siegel methodology, which is one of the most popular parametric yield curve estimation models.

Using the data for currency swap markets for various maturities, we provide zero-coupon swap yield curve for the period between January 2011 and December 2017. The estimation results indicate that there are no systematic deviations between fitted and quoted currency swap rates although some small differences exist in the very short-end of the curve. The in-sample fit of Nelson Siegel methodology across different maturities shows that the mean absolute error of the currency swap rates is on average six basis points. The short end of the currency swap curve tends to have higher errors. Lastly, it is found out that the zero-coupon swap rates and currency forward implied rates tends to move together. This result is intuitive given the similarity of both products in nature. This finding points out the consistency of the yield curve estimation for currency swap rates. Additionally, this result shows that the prices in FX forward and currency swap markets do not deviate from no-arbitrage condition, which shows the efficiency in these offshore markets. Overall, this

study contributes to the literature by obtaining zero-coupon swap yield curve, which can be safely used by investors and regulatory authorities.

References

AKINCI. Özge. Burcu GURCIHAN, Refet GURKAYNAK and Özgür OZEL; (2006), "Devlet Ic Borclanma Senetleri Icin Getiri Egrisi Tahmini (in Turkish)", Central Bank Working Paper, 06/08.

BIS, (2005), Zero-Coupon Yield Curves:Technical Documentation, BIS Papers No. 25.

CEPNI, Oğuzhan and Doruk KUCUKSARAC; (2017), "Optimal Mix of the Extended Nelson Siegel Model for Turkish Sovereign Yield Curve", Economics Bulletin, 37(2): pp. 1133-1142.

KUCUKSARAC, Doruk; (2017), "A New Approach for Turkish Term Structure: Cubic B-Spline Basis with Variable Roughness Penalty", Central Bank Research Notes, 11/17.

NELSON, Charles R. and Andrew F. SIEGEL; (1987), "Parsimonious modeling of yield curves", Journal of business, 60(4): pp. 473-489.