



On $\delta\alpha$, δp and δs -Irresolute Functions

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Abstract. In this paper is to introduce and investigate new classes of various irresolute functions and obtain some of their properties in topological spaces.

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1. Introduction

Recall the concepts of α -open [26] (resp. semi-open [15], preopen [18], β -open [11]) sets and semi α -irresolute [2] (resp. semi α -preirresolute [3]) functions in topological spaces.

The main purpose of this paper is to define and study the notions of new classes of functions, namely $\delta\alpha$ -irresolute, δp -irresolute and δs -irresolute functions, and to give some properties of these functions in topological spaces.

2. Preliminaries

Throughout this paper, spaces always mean topological spaces and $f : X \rightarrow Y$ denotes a single valued function of a space (X, τ) into a space (Y, σ) . Let S be a subset of a space (X, τ) . The closure and the interior of S are denoted by $Cl(S)$ and $Int(S)$, respectively.

Here we recall the following known definitions and properties.

Definition 1. A subset S of a space (X, τ) is said to be α -open [26] (resp. semi-open [15], preopen [18], β -open [11]) if $S \subset Int(Cl(Int(S)))$ (resp. $S \subset Cl(Int(S))$, $S \subset Int(Cl(S))$, $S \subset Cl(Int(Cl(S)))$).

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A point $x \in X$ is called the δ -cluster point of A if $A \cap \text{Int}(Cl(U)) \neq \emptyset$ for every open set U of X containing x . The set of all δ -cluster points of A is called the δ -cluster of A , denoted by $Cl_\delta(A)$. A subset A of X is called δ -closed [27] if $A = Cl_\delta(A)$. The complement of a δ -closed set is called δ -open [27]. A subset A of X is said to be a δ -semiopen [23] if there exists a δ -open set U of X such that $U \subset A \subset Cl(U)$. The complement of a δ -semiopen set is called δ -semiclosed set. A point $x \in X$ is called the δ -semicluster point of A if $A \cap U \neq \emptyset$ for every δ -semiopen set U of X containing x . The set of all δ -semicluster points of A is called the δ -semiclosure [23] of A , denoted by $\delta Cl_s(A)$. The family of all α -open (resp. semi-open, preopen, β -open, δ -open, δ -semiopen) sets in a space (X, τ) is denoted by $\tau^\alpha = \alpha(X)$ (resp. $SO(X)$, $PO(X)$, $\beta O(X)$, $\delta O(X)$, $\delta SO(X)$). It is shown in [26] that τ^α is a topology for X . Moreover, $\tau \subset \tau^\alpha = PO(X) \cap SO(X) \subset \beta O(X)$.

The complement of an α -open (resp. preopen, semi-open) set is said to be α -closed [17] (resp. preclosed [18], semi-closed [8]). The intersection of all α -closed (resp. preclosed, semi-closed) sets in (X, τ) containing a subset A is called the α -closure [17] (resp. preclosure [10], semi-closure [8]) of A , denoted by $\alpha Cl(A)$ (resp. $pCl(A)$, $sCl(A)$).

The union of all α -open (resp. preopen, semi-open, δ -open) sets of X contained in A is called the α -interior [1] (resp. preinterior [19], semi-interior [8], δ -interior [27]) of A and is denoted by $\alpha Int(A)$ (resp. $pInt(A)$, $sInt(A)$, $Int_\delta(A)$).

A subset S of a space (X, τ) is δ -semiopen [23] (resp. δ -semiclosed) if $S \subset Cl(Int_\delta(S))$ (resp. $Int(Cl_\delta(S)) \subset S$).

Lemma 1 (Park et al. [23]). *The intersection (resp. union) of arbitrary collection of δ -semiclosed (resp. δ -semiopen) sets in (X, τ) is δ -semiclosed (resp. δ -semiopen). And $A \subset X$ is δ -semiclosed if and only if $A = \delta Cl_s(A)$.*

Lemma 2 ([7, 10, 22, 14]). *Let $\{X_\lambda : \lambda \in \Lambda\}$ be any family of topological spaces and U_{λ_i} be a nonempty subset of X_{λ_i} for each $i = 1, 2, \dots, n$. Then $U = \prod_{\lambda \neq \lambda_i} X_\lambda \times \prod_{i=1}^n U_{\lambda_i}$ is a nonempty α -open [7] (resp. preopen [10], semi-open [22], δ -semiopen [14]) subset of $\prod X_\lambda$ if and only if U_{λ_i} is α -open (resp. preopen, semi-open, δ -semiopen) in X_{λ_i} for each $i = 1, 2, \dots, n$.*

Lemma 3. *Let A and B be subsets of a space (X, τ) . Then we have*

- (1) *If $A \in \delta SO(X)$ and $B \in \delta O(X)$, then $A \cap B \in \delta SO(B)$ [14].*
- (2) *If $A \in \delta SO(B)$ and $B \in \delta O(X)$, then $A \in \delta SO(X)$ [6].*

Definition 2. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be:*

- *semi α -irresolute [2] if $f^{-1}(V)$ is semi-open set in X for every α -open subset V of Y .*
- *semi α -preirresolute [3] if $f^{-1}(V)$ is semi-open set in X for every preopen subset V of Y .*
- *(δ, β) -irresolute [6] if $f^{-1}(V)$ is δ -semiopen set in X for every β -open subset V of Y .*
- *δ -semi-continuous [25] if $f^{-1}(V)$ is δ -semiopen set in X for every open subset V of Y .*
- *α -irresolute [17] if $f^{-1}(V)$ is α -open set in X for every α -open subset V of Y .*

- *preirresolute* [24] if $f^{-1}(V)$ is preopen set in X for every preopen subset V of Y .
- *irresolute* [9] if $f^{-1}(V)$ is semi-open set in X for every semi-open subset V of Y .

3. $\delta\alpha$, δp and δs -Irresolute Functions

Definition 3. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $\delta\alpha$ -irresolute, δp -irresolute and δs -irresolute if $f^{-1}(V)$ is δ -semiopen set in X for every α -open (resp. preopen, semi-open) subset V of Y .

From the definitions, we have the following relationships:

$$\begin{array}{ccccc}
 (\delta, \beta) - \text{irresoluteness} & \rightarrow & \delta p - \text{irresoluteness} & \rightarrow & \text{semi} - \alpha - \text{preirresoluteness} \\
 \downarrow & & \downarrow & & \downarrow \\
 \delta s - \text{irresoluteness} & \rightarrow & \delta \alpha - \text{irresoluteness} & \rightarrow & \text{semi} - \alpha - \text{irresoluteness}
 \end{array}$$

However the converses of the above implications are not true in general by the following examples.

Example 1. Let $X = \{a, b, c\}$ with topologies $\tau = \{X, \emptyset, \{a, b\}\}$ and $\sigma = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let a function $f : (X, \tau) \rightarrow (X, \sigma)$ be defined by $f(a) = f(b) = a$ and $f(c) = c$. Then f is semi α -preirresolute and hence semi α -irresolute but it is neither $\delta\alpha$ -irresolute, δs -irresolute nor δp -irresolute.

Example 2. Let $X = \{a, b, c, d\}$ with topologies $\tau = \{X, \emptyset, \{a, b, c\}\}$ and $\sigma = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let a function $f : (X, \tau) \rightarrow (X, \sigma)$ be defined by $f(a) = f(b) = f(c) = b$ and $f(d) = c$. Then f is δs -irresolute and hence $\delta\alpha$ -irresolute but it is not semi α -preirresolute.

Theorem 1. For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent:

- f is $\delta\alpha$ -irresolute;
- $f : (X, \tau) \rightarrow (Y, \sigma^\alpha)$ is δ -semi-continuous;
- For each $x \in X$ and each α -open set V of Y containing $f(x)$, there exists a δ -semiopen set U of X containing x such that $f(U) \subset V$;
- $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$ for every α -open set V of Y ;
- $f^{-1}(F)$ is δ -semiclosed in X for every α -closed set F of Y ;
- $Int(Cl_\delta(f^{-1}(B))) \subset f^{-1}(\alpha Cl(B))$ for every subset B of Y ;
- $f(Int(Cl_\delta(A))) \subset \alpha Cl(f(A))$ for every subset A of X .

Proof. (a) $\Rightarrow$ (b). Let $x \in X$ and V be any α -open set of Y containing $f(x)$. By Definition 3, $f^{-1}(V) \in \delta SO(X)$ containing x and hence $f : (X, \tau) \rightarrow (Y, \sigma^\alpha)$ is δ -semi-continuous.

(b) $\Rightarrow$ (c). Let $x \in X$ and V be any α -open set of Y containing $f(x)$. Set $U = f^{-1}(V)$, then by (b), U is a δ -semiopen set of X containing x and $f(U) \subset V$.

(c) $\Rightarrow$ (d). Let V be any α -open subset of Y and $x \in f^{-1}(V)$. By (c), there exists a δ -semiopen set U of X containing x such that $f(U) \subset V$. Therefore, we obtain $x \in U \subset Cl(Int_\delta(U)) \subset Cl(Int_\delta(f^{-1}(V)))$ and hence $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$.

(d) $\Rightarrow$ (e). Let F be any α -closed subset of Y . Set $V = Y - F$, then V is α -open in Y . By (d), we have $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$ and hence $f^{-1}(F) = X - (f^{-1}(Y - F)) = X - f^{-1}(V)$ is δ -semiclosed in X .

(e) $\Rightarrow$ (f). Let B be any subset of Y . Since $\alpha Cl(B)$ is α -closed in Y , $f^{-1}(\alpha Cl(B))$ is δ -semiclosed in X and hence $Int(Cl_\delta(f^{-1}(\alpha Cl(B)))) \subset f^{-1}(\alpha Cl(B))$. Thus we have $Int(Cl_\delta(f^{-1}(B))) \subset f^{-1}(\alpha Cl(B))$.

(f) $\Rightarrow$ (g). Let A be any subset of X . By (f), we obtain $Int(Cl_\delta(A)) \subset Int(Cl_\delta(f^{-1}(f(A)))) \subset f^{-1}(\alpha Cl(f(A)))$ and hence $f(Int(Cl_\delta(A))) \subset \alpha Cl(f(A))$.

(g) $\Rightarrow$ (a). Let V be any α -open subset of Y . Since $f^{-1}(Y - V) = X - f^{-1}(V)$ is a subset of X and by (g), we obtain

$$f(Int(Cl_\delta(f^{-1}(Y - V)))) \subset \alpha Cl(f(f^{-1}(Y - V))) \subset \alpha Cl(Y - V) = Y - \alpha Int(V) = Y - V$$

and hence

$$\begin{aligned} X - Cl(Int_\delta(f^{-1}(V))) &= Int(Cl_\delta(X - f^{-1}(V))) \\ &= Int(Cl_\delta(f^{-1}(Y - V))) \subset f^{-1}(f(Int(Cl_\delta(f^{-1}(Y - V))))) \subset f^{-1}(Y - V) \\ &= X - f^{-1}(V). \end{aligned}$$

Therefore, we have $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$ and hence $f^{-1}(V)$ is δ -semiopen in X . Thus the function f is $\delta\alpha$ -irresolute.

Now, the proofs of the following two theorems are similar to Theorem 1 and are thus omitted.

Theorem 2. The following are equivalent for a function $f : (X, \tau) \rightarrow (Y, \sigma)$:

- (a) f is δp -irresolute;
- (b) For each $x \in X$ and each preopen set V of Y containing $f(x)$, there exists a δ -semiopen set U of X containing x such that $f(U) \subset V$;
- (c) $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$ for every preopen set V of Y ;
- (d) $f^{-1}(F)$ is δ -semiclosed in X for every preclosed set F of Y ;
- (e) $Int(Cl_\delta(f^{-1}(B))) \subset f^{-1}(pCl(B))$ for every subset B of Y ;
- (f) $f(Int(Cl_\delta(A))) \subset pCl(f(A))$ for every subset A of X .

Theorem 3. For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent:

- (a) f is δs -irresolute;
- (b) For each $x \in X$ and each semi-open set V of Y containing $f(x)$, there exists a δ -semiopen set U of X containing x such that $f(U) \subset V$;
- (c) $f^{-1}(V) \subset Cl(Int_{\delta}(f^{-1}(V)))$ for every semi-open set V of Y ;
- (d) $f^{-1}(F)$ is δ -semiclosed in X for every semi-closed set F of Y ;
- (e) $Int(Cl_{\delta}(f^{-1}(B))) \subset f^{-1}(sCl(B))$ for every subset B of Y ;
- (f) $f(Int(Cl_{\delta}(A))) \subset sCl(f(A))$ for every subset A of X .

The proofs of the other parts of the following theorems follow by a similar way and are thus omitted.

Theorem 4. Let $f : X \rightarrow Y$ be a function and $g : X \rightarrow X \times Y$ the graph function, given by $g(x) = (x, f(x))$ for every $x \in X$. Then f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) if g is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute).

Proof. Let $x \in X$ and V be any α -open (resp. preopen, semi-open) set of Y containing $f(x)$. Then, by Lemma 2, The set $X \times V$ is α -open (resp. preopen, semi-open) in $X \times Y$ containing $g(x)$. Since g is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), there exists a δ -semiopen set U of X containing x such that $g(U) \subset X \times V$ and hence $f(U) \subset V$. Thus f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute).

Theorem 5. If a function $f : X \rightarrow \Pi Y_{\lambda}$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), then $P_{\lambda} \circ f : X \rightarrow Y_{\lambda}$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) for each $\lambda \in \Lambda$, where P_{λ} is the projection of ΠY_{λ} onto Y_{λ} .

Proof. Let V_{λ} be any α -open (resp. preopen, semi-open) set of Y_{λ} . Since P_{λ} is continuous and open, it is α -irresolute [20, Theorem 3.2] (resp. preirresolute [20, Theorem 3.4], irresolute [9, Theorem 1.2]) and hence $P_{\lambda}^{-1}(V_{\lambda})$ is α -open (resp. preopen, semi-open) in ΠY_{λ} . Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), then $f^{-1}(P_{\lambda}^{-1}(V_{\lambda})) = (P_{\lambda} \circ f)^{-1}(V_{\lambda})$ is δ -semiopen in X . Hence $P_{\lambda} \circ f$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) for each $\lambda \in \Lambda$.

Theorem 6. If the product function $f : \Pi X_{\lambda} \rightarrow \Pi Y_{\lambda}$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), then $f_{\lambda} : X_{\lambda} \rightarrow Y_{\lambda}$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) for each $\lambda \in \Lambda$.

Proof. Let $\lambda_0 \in \Lambda$ be an arbitrary fixed index and V_{λ_0} be any α -open (resp. preopen, semi-open) set of Y_{λ_0} . Then $\Pi Y_{\lambda} \times V_{\lambda_0}$ is α -open (resp. preopen, semi-open) in ΠY_{λ} by Lemma 2, where $\lambda_0 \neq \lambda \in \Lambda$. Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), then $f^{-1}(\Pi Y_{\lambda} \times V_{\lambda_0}) = \Pi X_{\lambda} \times f_{\lambda_0}^{-1}(V_{\lambda_0})$ is δ -semiopen in ΠX_{λ} and hence, by Lemma 2, $f_{\lambda_0}^{-1}(V_{\lambda_0})$ is δ -semiopen in X_{λ_0} . This implies that f_{λ_0} is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute).

Theorem 7. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) and A is a δ -open subset of X , then the restriction $f|_A : A \rightarrow Y$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute).

Proof. Let V be any α -open (resp. preopen, semi-open) set of Y . Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), then $f^{-1}(V)$ is δ -semiopen in X . Since A is δ -open in X , $(f|_A)^{-1}(V) = A \cap f^{-1}(V)$ is δ -semiopen in A by the condition (1) of Lemma 3. Hence $f|_A$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute).

Theorem 8. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function and $\{A_\lambda : \lambda \in \Lambda\}$ be a cover of X by δ -open sets of (X, τ) . Then f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) if $f|_{A_\lambda} : A_\lambda \rightarrow Y$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) for each $\lambda \in \Lambda$.

Proof. Let V be any α -open (resp. preopen, semi-open) set of Y . Since $f|_{A_\lambda}$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), then $(f|_{A_\lambda})^{-1}(V) = f^{-1}(V) \cap A_\lambda$ is δ -semiopen in A_λ . Since A_λ is δ -open in X , by the condition (2) of Lemma 3, $(f|_{A_\lambda})^{-1}(V)$ is δ -semiopen in X for $\lambda \in \Lambda$. Therefore $f^{-1}(V) = X \cap f^{-1}(V) = \cup \{A_\lambda \cap f^{-1}(V) : \lambda \in \Lambda\} = \cup \{(f|_{A_\lambda})^{-1}(V) : \lambda \in \Lambda\}$ is δ -semiopen in X because the union of δ -semiopen sets is a δ -semiopen set by Lemma 1. Hence f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute).

Theorem 9. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be functions. Then the composition $g \circ f : X \rightarrow Z$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) if f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) and g is α -irresolute (resp. preirresolute, irresolute).

Proof. Let W be any α -open (resp. preopen, semi-open) subset of Z . Since g is α -irresolute (resp. preirresolute, irresolute), $g^{-1}(W)$ is α -open (resp. preopen, semi-open) in Y . Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is δ -semiopen in X and hence $g \circ f$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute).

We recall that a space (X, τ) is said to be submaximal [4] if every dense subset of X is open in X and extremally disconnected [26] if the closure of each open subset of X is open in X . The following theorem follows from the fact that if (X, τ) is a submaximal and extremally disconnected space, then $\tau = \tau^\alpha = SO(X) = PO(X) = \beta O(X)$ [12, 21].

Theorem 10. Let (Y, σ) be a submaximal and extremally disconnected space and let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then we have

$$\delta\alpha\text{-irresoluteness} \Leftrightarrow \delta p\text{-irresoluteness} \Leftrightarrow \delta s\text{-irresoluteness} \Leftrightarrow (\delta, \beta)\text{-irresoluteness}.$$

Recall that a topological space (X, τ) is called $\alpha - T_2$ [17] (resp. pre- T_2 [13], semi- T_2 [16], δ -semi- T_2 [5] if for any distinct pair of points x and y in X , there exist $U \in \alpha(X, x)$ and $V \in \alpha(X, y)$ (resp. $U \in PO(X, x)$ and $V \in PO(X, y)$, $U \in SO(X, x)$ and $V \in SO(X, y)$, $U \in \delta SO(X, x)$ and $V \in \delta SO(X, y)$) such that $U \cap V = \emptyset$.

Theorem 11. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) injection and Y is $\alpha - T_2$ (resp. pre- T_2 , semi- T_2), then X is δ -semi- T_2 .

Proof. Let x and y be distinct points of X . Then $f(x) \neq f(y)$. Since Y is $\alpha - T_2$ (resp. pre- T_2 , semi- T_2), there exist disjoint α -open (resp. preopen, semi-open) sets V and W containing $f(x)$ and $f(y)$, respectively. Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), there exist δ -semiopen sets U and H containing x and y , respectively, such that $f(U) \subset V$ and $f(H) \subset W$. It follows that $U \cap H = \emptyset$. This shows that X is δ -semi- T_2 .

Lemma 4 (Lee et. al. [14]). *If A_i is a δ -semiopen set of X_i ($i = 1, 2$), then $A_1 \times A_2$ is δ -semiopen in $X_1 \times X_2$.*

Theorem 12. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) and Y is $\alpha - T_2$ (resp. pre- T_2 , semi- T_2), then the set $E = \{(x, y) : f(x) = f(y)\}$ is δ -semiclosed in $X \times X$.*

Proof. Suppose that $(x, y) \notin E$. Then $f(x) \neq f(y)$. Since Y is $\alpha - T_2$ (resp. pre- T_2 , semi- T_2), there exist $V \in \alpha(Y, f(x))$ and $W \in \alpha(Y, f(y))$ (resp. $V \in PO(Y, f(x))$ and $W \in PO(Y, f(y))$, $V \in SO(Y, f(x))$ and $W \in SO(Y, f(y))$) such that $V \cap W = \emptyset$. Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), there exist δ -semiopen sets U and H containing x and y , respectively, such that $f(U) \subset V$ and $f(H) \subset W$. Set $G = U \times H$. By Lemma 4, $(x, y) \in G \in \delta SO(X \times X)$ and $G \cap E = \emptyset$. This means that $\delta Cl_s(E) \subset E$ and hence the set E is δ -semiclosed in $X \times X$.

Definition 4. *For a function $f : X \rightarrow Y$, the graph $G(f) = \{(x, f(x)) : x \in X\}$ is called $\delta\alpha$ -closed (resp. δp -closed, δs -closed) if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in \delta SO(X, x)$ and $V \in \alpha(Y, y)$ (resp. $V \in PO(Y, y)$, $V \in SO(Y, y)$) such that $(U \times V) \cap G(f) = \emptyset$.*

Theorem 13. *If $f : X \rightarrow Y$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) and Y is $\alpha - T_2$ (resp. pre- T_2 , semi- T_2), then $G(f)$ is $\delta\alpha$ -closed (resp. δp -closed, δs -closed) in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. This implies that $f(x) \neq y$. Since Y is $\alpha - T_2$ (resp. pre- T_2 , semi- T_2), there exist disjoint α -open (resp. preopen, semi-open) sets V and W in Y containing $f(x)$ and y , respectively. Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), there exists a δ -semiopen set U of X containing x such that $f(U) \subset V$. Therefore $f(U) \cap W = \emptyset$ and hence $(U \times W) \cap G(f) = \emptyset$. Thus $G(f)$ is $\delta\alpha$ -closed (resp. δp -closed, δs -closed) in $X \times Y$.

Theorem 14. *If $f : X \rightarrow Y$ is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute) injection with a $\delta\alpha$ -closed (resp. δp -closed, δs -closed) graph, then X is δ -semi- T_2 .*

Proof. Let x and y be any distinct points of X . Then $f(x) \neq f(y)$ and hence $(x, f(y)) \in (X \times Y) - G(f)$. Since $G(f)$ is $\delta\alpha$ -closed (resp. δp -closed, δs -closed), there exist $U \in \delta SO(X, x)$ and $V \in \alpha(Y, f(y))$ (resp. $V \in PO(Y, f(y))$, $V \in SO(Y, f(y))$) such that $(U \times V) \cap G(f) = \emptyset$ and hence $f(U) \cap V = \emptyset$. Since f is $\delta\alpha$ -irresolute (resp. δp -irresolute, δs -irresolute), there exists $G \in \delta SO(X, y)$ such that $f(G) \subset V$. Thus we have $f(U) \cap f(G) = \emptyset$ and hence $U \cap G = \emptyset$. This shows that X is δ -semi- T_2 .

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