

PICTURE FUZZY INTERIOR ANTIIDEALS

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ABSTRACT. This paper introduces picture fuzzy interior antiideals in semi-groups and investigates their properties. Their relationships with classical interior antiideals are characterized by means of characteristic functions and level subsets. The paper also establishes results on intersections, level subsets, images under isomorphisms, inverse images under homomorphisms, and Cartesian products.

1. INTRODUCTION

The concept of fuzzy sets, introduced by Zadeh [16], provides a flexible mathematical framework for dealing with uncertainty and vagueness. Unlike classical sets, each element in a fuzzy set is associated with a membership degree between 0 and 1. Rosenfeld [13] extended this idea to algebraic structures by defining fuzzy subgroups and fuzzy ideals, which initiated extensive research on fuzzy algebraic systems and their applications in modeling approximate reasoning.

In semigroup theory, several subclasses such as ideals, filters, and antiideals have been introduced to describe internal structural properties. The notion of antiideals, first introduced by Schwarz [14] and further studied by Iseki [7], has been used to characterize regular and intra-regular semigroups.

Atanassov [5] (1986) introduced intuitionistic fuzzy sets by adding a non-membership degree to Zadeh's framework. Later, Cuong [6] (2013) extended this framework to picture fuzzy sets (PFSs) by introducing a neutral membership degree, which allows positive, neutral, and negative responses to be represented.

Cuong initially considered the type-1 order relation $\leq_1$ (type-1) on $\mathbb{D}^*$, defined as follows:

$$(x_1, x_2, x_3) \leq_1 (y_1, y_2, y_3) \text{ iff } x_1 \leq y_1, x_2 \leq y_2, x_3 \geq y_3.$$

Subsequently, picture fuzzy set theory was applied to various algebraic structures. For example, Panpho and Yiarayong [11] studied picture fuzzy ideals in commutative rings, focusing on intersection and product operations and their basic properties. Yiarayong [15] characterized regular and intra-regular semigroups by means

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of picture fuzzy bi-ideals. Adak et al. [1] studied semiprime ideals in the picture fuzzy setting. However, some operations defined for picture fuzzy sets in these studies—notably intersections and products—may fail to produce a picture fuzzy set. The same issue can arise for characteristic functions. Moreover, $\mathbb{D}^*$ is not a complete lattice with respect to the type-1 order relation $\leq_1$. To address these issues, Okumuş et al. [9] reconsidered ideals in semigroups using a different order (type-3) on D^* .

Antiideals and bi antiideals, together with their fuzzy counterparts, were investigated in [2]. Generalized bi antiideals and their fuzzy counterparts were considered in [4]. Al Tahan and Hoskova-Mayerova [3] (2025) introduced fuzzy interior antiideals and fuzzy m -interior antiideals, extending the study of antiideals to the fuzzy setting. Generalized (T, S_T) -intuitionistic fuzzy antiideals on semigroups were investigated in [12], and picture fuzzy left and right antiideals were studied in [10]. The present paper focuses on picture fuzzy interior antiideals and establishes their characterizations in terms of picture characteristic functions and level subsets, together with their intersection, mapping, and Cartesian product properties.

The structure of the paper is as follows. Section 2 presents basic definitions and preliminaries. Section 3 introduces picture fuzzy interior antiideals and discusses their properties, level-set characterizations, and relationships with classical antiideals. Section 4 concludes the paper and presents possible directions for future research.

2. PRELIMINARIES

Definition 2.1. Let X be a non-empty set. A fuzzy set of X is a function $\alpha_A : X \rightarrow [0, 1]$,

$$A = \{(p, \alpha_A(p)) : p \in X\}$$

Definition 2.2. Let $X \neq \emptyset$. An intuitionistic fuzzy set (IFS) A on a nonempty set X is given by

$$A = \{(p, \alpha_A(p), \beta_A(p)) : p \in X\},$$

where $\alpha_A, \beta_A : X \rightarrow [0, 1]$ are the membership and non-membership functions, respectively, and

$$0 \leq \alpha_A(p) + \beta_A(p) \leq 1$$

for all $p \in X$.

Cuong [6] introduced picture fuzzy sets by adding a neutral membership degree to intuitionistic fuzzy sets. This framework allows uncertainty to be represented more flexibly, particularly in contexts such as voting, where responses may be positive, neutral, or negative.

Cuong [6] has considered the set.

$$\mathbb{D}^* = \{(p_1, p_2, p_3) \in [0, 1]^3 : p_1 + p_2 + p_3 \leq 1\}.$$

A picture fuzzy set on X is a mapping $A : X \rightarrow D^*$ of the form

$$A = \{(p, \alpha_A(p), \beta_A(p), \gamma_A(p)) : p \in X\},$$

where $\alpha_A(p)$, $\beta_A(p)$, and $\gamma_A(p)$ denote the membership, neutral-membership, and non-membership degrees of p , respectively, and

$$0 \leq \alpha_A(p) + \beta_A(p) + \gamma_A(p) \leq 1$$

for all $p \in X$. The family of all picture fuzzy sets on X is denoted by $PFS(X)$. For $\mathbf{p} = (p_1, p_2, p_3), \mathbf{r} = (r_1, r_2, r_3) \in \mathbb{D}^*$, the type-3 order $\leq_3$ on $\mathbb{D}^*$ is defined by [8]

$$\mathbf{p} \leq_3 \mathbf{r} \quad \text{if and only if} \quad p_1 \leq r_1, \quad p_1 + p_2 \leq r_1 + r_2, \quad p_3 \geq r_3.$$

Definition 2.3. [9] Let $\mathcal{P}, \mathcal{R} \in PFS(X)$. The relation $\mathcal{P} \subseteq \mathcal{R}$ is defined by the following conditions for all $p \in X$:

$$\begin{aligned} \alpha_{\mathcal{P}}(p) &\leq \alpha_{\mathcal{R}}(p), \\ \alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p) &\leq \alpha_{\mathcal{R}}(p) + \beta_{\mathcal{R}}(p), \\ \gamma_{\mathcal{P}}(p) &\geq \gamma_{\mathcal{R}}(p). \end{aligned}$$

Definition 2.4. [9] Let $\mathcal{P}, \mathcal{R} \in PFS(X)$. Their intersection and union are defined as follows for all $p \in X$:

$$\begin{aligned} \alpha_{\mathcal{P} \cap \mathcal{R}}(p) &= \alpha_{\mathcal{P}}(p) \wedge \alpha_{\mathcal{R}}(p), \\ \beta_{\mathcal{P} \cap \mathcal{R}}(p) &= ((\alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)) \wedge (\alpha_{\mathcal{R}}(p) + \beta_{\mathcal{R}}(p))) - (\alpha_{\mathcal{P}}(p) \wedge \alpha_{\mathcal{R}}(p)), \\ \gamma_{\mathcal{P} \cap \mathcal{R}}(p) &= \gamma_{\mathcal{P}}(p) \vee \gamma_{\mathcal{R}}(p), \\ \alpha_{\mathcal{P} \cup \mathcal{R}}(p) &= \alpha_{\mathcal{P}}(p) \vee \alpha_{\mathcal{R}}(p), \\ \beta_{\mathcal{P} \cup \mathcal{R}}(p) &= ((\alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)) \vee (\alpha_{\mathcal{R}}(p) + \beta_{\mathcal{R}}(p))) - (\alpha_{\mathcal{P}}(p) \vee \alpha_{\mathcal{R}}(p)), \\ \gamma_{\mathcal{P} \cup \mathcal{R}}(p) &= \gamma_{\mathcal{P}}(p) \wedge \gamma_{\mathcal{R}}(p). \end{aligned}$$

These are called the intersection and union of $\mathcal{P}$ and $\mathcal{R}$, respectively.

Definition 2.5. [9] Let $\{\mathcal{P}_t \mid t \in T\} \subseteq PFS(X)$. The intersection and union of this family are defined as follows for all $p \in X$:

$$\begin{aligned} \alpha_{\bigcap_{t \in T} \mathcal{P}_t}(p) &= \bigwedge_{t \in T} \alpha_{\mathcal{P}_t}(p), \\ \beta_{\bigcap_{t \in T} \mathcal{P}_t}(p) &= \bigwedge_{t \in T} (\alpha_{\mathcal{P}_t}(p) + \beta_{\mathcal{P}_t}(p)) - \bigwedge_{t \in T} \alpha_{\mathcal{P}_t}(p), \\ \gamma_{\bigcap_{t \in T} \mathcal{P}_t}(p) &= \bigvee_{t \in T} \gamma_{\mathcal{P}_t}(p), \\ \alpha_{\bigcup_{t \in T} \mathcal{P}_t}(p) &= \bigvee_{t \in T} \alpha_{\mathcal{P}_t}(p), \\ \beta_{\bigcup_{t \in T} \mathcal{P}_t}(p) &= \bigvee_{t \in T} (\alpha_{\mathcal{P}_t}(p) + \beta_{\mathcal{P}_t}(p)) - \bigvee_{t \in T} \alpha_{\mathcal{P}_t}(p), \\ \gamma_{\bigcup_{t \in T} \mathcal{P}_t}(p) &= \bigwedge_{t \in T} \gamma_{\mathcal{P}_t}(p). \end{aligned}$$

These are called the intersection and union of the family $\{\mathcal{P}_t \mid t \in T\}$, respectively.

Definition 2.6. [9] Let $\mathcal{P} \in PFS(X)$ and $\mathcal{R} \in PFS(X')$. The Cartesian product $\mathcal{P} \times \mathcal{R} \in PFS(X \times X')$ is defined as follows:

$$\begin{aligned} \alpha_{\mathcal{P} \times \mathcal{R}}(p, r) &= \alpha_{\mathcal{P}}(p) \wedge \alpha_{\mathcal{R}}(r), \\ \beta_{\mathcal{P} \times \mathcal{R}}(p, r) &= (\alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)) \wedge (\alpha_{\mathcal{R}}(r) + \beta_{\mathcal{R}}(r)) - (\alpha_{\mathcal{P}}(p) \wedge \alpha_{\mathcal{R}}(r)), \\ \gamma_{\mathcal{P} \times \mathcal{R}}(p, r) &= \gamma_{\mathcal{P}}(p) \vee \gamma_{\mathcal{R}}(r). \end{aligned}$$

for all $(p, r) \in X \times X'$. The picture fuzzy set $\mathcal{P} \times \mathcal{R}$ is called the Cartesian product of $\mathcal{P}$ and $\mathcal{R}$.

Definition 2.7. [9] Let $A \subseteq X$. Then the *picture characteristic function* of A , denoted by $C^A = (\alpha_{C^A}, \beta_{C^A}, \gamma_{C^A})$ is defined as:

$$\begin{aligned}\alpha_{C^A}(p) &= \begin{cases} 1, & p \in A \\ 0, & \text{otherwise.} \end{cases} \\ \beta_{C^A}(p) &= \begin{cases} 0, & p \in A \\ 0, & \text{otherwise.} \end{cases} \\ \gamma_{C^A}(p) &= \begin{cases} 0, & p \in A \\ 1, & \text{otherwise.} \end{cases}\end{aligned}$$

Definition 2.8. [9] Let f be a mapping from X into X' and $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(X)$, $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(X')$. Then the *image* of $\mathcal{P}$ and the *pre-image* of $\mathcal{R}$ under f are denoted by $f(\mathcal{P})$ and $f^{-1}(\mathcal{R})$, respectively and are defined as: for all $p \in X$ and $p' \in X'$,

$$\begin{aligned}\alpha_{f(\mathcal{P})}(p') &= \bigvee_{p'=f(p)} \alpha_{\mathcal{P}}(p), \\ \beta_{f(\mathcal{P})}(p') &= \bigvee_{p'=f(p)} (\alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)) - \bigvee_{p'=f(p)} \alpha_{\mathcal{P}}(p), \\ \gamma_{f(\mathcal{P})}(p') &= \bigwedge_{p'=f(p)} \gamma_{\mathcal{P}}(p), \\ \alpha_{f^{-1}(\mathcal{R})}(p) &= \alpha_{\mathcal{R}}(f(p)), \\ \beta_{f^{-1}(\mathcal{R})}(p) &= \beta_{\mathcal{R}}(f(p)), \\ \gamma_{f^{-1}(\mathcal{R})}(p) &= \gamma_{\mathcal{R}}(f(p)).\end{aligned}$$

Definition 2.9. [9] Let $\mathcal{P} \in PFS(X)$. For each $\alpha \in [0, 1]$, define the following level subsets of $\mathcal{P}$:

$$\begin{aligned}PM(\mathcal{P} : \alpha) &= \{p \in X \mid \alpha \leq \alpha_{\mathcal{P}}(p)\} \\ N(\mathcal{P} : \alpha) &= \{p \in X \mid \alpha \leq \alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)\} \\ NM(\mathcal{P} : \alpha) &= \{p \in X \mid \alpha \geq \gamma_{\mathcal{P}}(p)\}\end{aligned}$$

A semigroup is a nonempty set S equipped with an associative binary operation. Unless otherwise stated, S is assumed to be a semigroup throughout this paper. The *product* of two subsets A and B of a semigroup S is defined as

$$AB = \{ab \mid a \in A, b \in B\}.$$

Definition 2.10. [3, 7] Let A be any non-empty subset of S . Then

- (i) A is called a *left antiideal* of S if $SA \cap A = \emptyset$,
- (ii) A is called a *right antiideal* of S if $AS \cap A = \emptyset$,
- (iii) A is called an *antiideal* of S if it is both a left antiideal and a right antiideal of S ,
- (iv) A is called an *interior antiideal* of S if $SAS \cap A = \emptyset$,

The sets of all left antiideals, right antiideals, antiideals, and interior antiideals of S are written as $LAI(S)$, $RAI(S)$, $AI(S)$, and $IAI(S)$, respectively.

Definition 2.11. [10] Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S)$. Then

(i) $\mathcal{P}$ is called a picture fuzzy left antiideal of S if, for all $p_1, p_2 \in S$,

$$\begin{aligned}\alpha_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{P}}(p_1 p_2) &= 0, \\ (\alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2)) \wedge (\alpha_{\mathcal{P}}(p_1 p_2) + \beta_{\mathcal{P}}(p_1 p_2)) &= 0, \\ \gamma_{\mathcal{P}}(p_2) \vee \gamma_{\mathcal{P}}(p_1 p_2) &= 1.\end{aligned}$$

(ii) $\mathcal{P}$ is called a picture fuzzy right antiideal of S if, for all $p_1, p_2 \in S$,

$$\begin{aligned}\alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_1 p_2) &= 0, \\ (\alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1)) \wedge (\alpha_{\mathcal{P}}(p_1 p_2) + \beta_{\mathcal{P}}(p_1 p_2)) &= 0, \\ \gamma_{\mathcal{P}}(p_1) \vee \gamma_{\mathcal{P}}(p_1 p_2) &= 1.\end{aligned}$$

3. PICTURE FUZZY INTERIOR ANTIIDEALS

Definition 3.1. Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S)$. Then $\mathcal{P}$ is a *picture fuzzy interior antiideal* of S if the following conditions hold for all $x, y, z \in S$:

$$\begin{aligned}\alpha_{\mathcal{P}}(xyz) \wedge \alpha_{\mathcal{P}}(y) &= 0, \\ (\alpha_{\mathcal{P}}(xyz) + \beta_{\mathcal{P}}(xyz)) \wedge (\alpha_{\mathcal{P}}(y) + \beta_{\mathcal{P}}(y)) &= 0, \\ \gamma_{\mathcal{P}}(xyz) \vee \gamma_{\mathcal{P}}(y) &= 1.\end{aligned}$$

The set of all picture fuzzy interior antiideals of S is denoted by $PFIAI(S)$.

Proposition 1. Let $A \subset S$ and $A \neq \emptyset$. $A \in IAI(S)$ if and only if $C^A \in PFIAI(S)$.

Proof. (i) Suppose first that $A \in IAI(S)$. Let $p_1, p_3 \in S$ and $p_2 \in A$. Since $SAS \cap A = \emptyset$, we have $p_1 p_2 p_3 \notin A$. Therefore,

$$\begin{aligned}\alpha_{C^A}(p_1 p_2 p_3) \wedge \alpha_{C^A}(p_2) &= 0 \\ (\alpha_{C^A}(p_1 p_2 p_3) + \beta_{C^A}(p_1 p_2 p_3)) \wedge (\alpha_{C^A}(p_2) + \beta_{C^A}(p_2)) &= 0, \\ \gamma_{C^A}(p_1 p_2 p_3) \vee \gamma_{C^A}(p_2) &= 1.\end{aligned}$$

Clearly, if $p_2 \notin A$

$$\begin{aligned}\alpha_{C^A}(p_1 p_2 p_3) \wedge \alpha_{C^A}(p_2) &= 0, \\ (\alpha_{C^A}(p_1 p_2 p_3) + \beta_{C^A}(p_1 p_2 p_3)) \wedge (\alpha_{C^A}(p_2) + \beta_{C^A}(p_2)) &= 0, \\ \gamma_{C^A}(p_1 p_2 p_3) \vee \gamma_{C^A}(p_2) &= 1.\end{aligned}$$

Therefore $C^A \in PFIAI(S)$.

Conversely, suppose that $C^A \in PFIAI(S)$. Assume, to the contrary, that $SAS \cap A \neq \emptyset$. Then there exist $p_1, p_3 \in S$ and $p_2 \in A$ such that $p_1 p_2 p_3 \in SAS \cap A$. This contradicts the defining condition of $\mathcal{P}$. Hence $SAS \cap A = \emptyset$, and therefore $A \in IAI(S)$. $\square$

Theorem 3.2. Let $\emptyset \neq T$ and let $\{\mathcal{P}_t \mid t \in T\} \subseteq PFIAI(S)$. Then $\bigcap_{t \in T} \mathcal{P}_t \in PFIAI(S)$.

Proof. For each $t \in T$, put $\delta_t(p) = \alpha_{\mathcal{P}_t}(p) + \beta_{\mathcal{P}_t}(p)$. By the definition of intersection,

$$\alpha_{\bigcap_{t \in T} \mathcal{P}_t}(p) = \bigwedge_{t \in T} \alpha_{\mathcal{P}_t}(p), \quad \alpha_{\bigcap_{t \in T} \mathcal{P}_t}(p) + \beta_{\bigcap_{t \in T} \mathcal{P}_t}(p) = \bigwedge_{t \in T} \delta_t(p),$$

and

$$\gamma_{\bigcap_{t \in T} \mathcal{P}_t}(p) = \bigvee_{t \in T} \gamma_{\mathcal{P}_t}(p).$$

Thus, for all $p_1, p_2, p_3 \in S$,

$$\alpha_{\bigcap_t \mathcal{P}_t}(p_1 p_2 p_3) \wedge \alpha_{\bigcap_t \mathcal{P}_t}(p_2) = \bigwedge_{t \in T} (\alpha_{\mathcal{P}_t}(p_1 p_2 p_3) \wedge \alpha_{\mathcal{P}_t}(p_2)) = 0,$$

$$(\alpha_{\bigcap_t \mathcal{P}_t}(p_1 p_2 p_3) + \beta_{\bigcap_t \mathcal{P}_t}(p_1 p_2 p_3)) \wedge (\alpha_{\bigcap_t \mathcal{P}_t}(p_2) + \beta_{\bigcap_t \mathcal{P}_t}(p_2)) = \bigwedge_{t \in T} (\delta_t(p_1 p_2 p_3) \wedge \delta_t(p_2)) = 0,$$

$$\gamma_{\bigcap_t \mathcal{P}_t}(p_1 p_2 p_3) \vee \gamma_{\bigcap_t \mathcal{P}_t}(p_2) = \bigvee_{t \in T} (\gamma_{\mathcal{P}_t}(p_1 p_2 p_3) \vee \gamma_{\mathcal{P}_t}(p_2)) = 1.$$

Therefore, $\bigcap_{t \in T} \mathcal{P}_t \in PFIAI(S)$. $\square$

The union of picture fuzzy interior antiideals may not be a picture fuzzy interior antiideal in general.

Example 3.3. Let $S = (\mathbb{N}^+, +)$, and let $\mathcal{P}$ and $\mathcal{R}$ be picture fuzzy sets on S defined below, where $A = \{1, 2\}$ and $B = \{3\}$, respectively.

$$\mathcal{P} = \begin{cases} (1, 0, 0) & p \in A \\ (0, 0, 1), & \text{otherwise.} \end{cases}$$

$$\mathcal{R} = \begin{cases} (1, 0, 0) & p \in B \\ (0, 0, 1), & \text{otherwise.} \end{cases}$$

It is readily verified that $\mathcal{P}, \mathcal{R} \in PFIAI(S)$. Since

$$\begin{aligned} \alpha_{\mathcal{P} \cup \mathcal{R}}(1 + 1 + 1) \wedge \alpha_{\mathcal{P} \cup \mathcal{R}}(1) &= (\alpha_{\mathcal{P}}(3) \vee \alpha_{\mathcal{R}}(3)) \wedge (\alpha_{\mathcal{P}}(1) \vee \alpha_{\mathcal{R}}(1)) \\ &= (0 \vee 1) \wedge (1 \vee 0) \\ &= 1 \\ &\neq 0 \end{aligned}$$

Thus, $\mathcal{P} \cup \mathcal{R} \notin PFIAI(S)$.

Theorem 3.4. Let $\mathcal{P} \in PFS(S)$. Then $\mathcal{P} \in PFIAI(S)$ if and only if, for every $\alpha \in (0, 1)$, each nonempty set among $PM(\mathcal{P} : \alpha)$, $N(\mathcal{P} : \alpha)$, and $NM(\mathcal{P} : \alpha)$ is an interior antiideal of S .

Proof. Assume first that $\mathcal{P} \in PFIAI(S)$, and let $\lambda \in (0, 1)$. If $p_2, p_1 p_2 p_3 \in PM(\mathcal{P} : \lambda)$, then

$$\alpha_{\mathcal{P}}(p_1 p_2 p_3) \wedge \alpha_{\mathcal{P}}(p_2) \geq \lambda > 0,$$

Therefore, every nonempty set $PM(\mathcal{P} : \lambda)$ is an interior antiideal of S . The same argument, applied to N and NM , shows that every nonempty level set of these types is also an interior antiideal of S .

Conversely, assume that all the non-empty level sets in the statement are interior antiideals of S . Let $p_1, p_2, p_3 \in S$. If

$$a = \alpha_{\mathcal{P}}(p_1 p_2 p_3) \wedge \alpha_{\mathcal{P}}(p_2) > 0,$$

then, taking $\lambda = a/2 \in (0, 1)$, we obtain $p_2, p_1 p_2 p_3 \in PM(\mathcal{P} : \lambda)$, a contradiction. Thus,

$$\alpha_{\mathcal{P}}(p_1 p_2 p_3) \wedge \alpha_{\mathcal{P}}(p_2) = 0.$$

The same level-set argument applied to N and NM yields the other two defining conditions. Therefore, $\mathcal{P} \in PFIAI(S)$. $\square$

Theorem 3.5. *Let $f : S \rightarrow S'$ be a semigroup isomorphism, and let $\mathcal{P} \in PFS(S)$. If $\mathcal{P} \in PFIAI(S)$, then $f(\mathcal{P}) \in PFIAI(S')$.*

Proof. Let $\mathcal{P} \in PFIAI(S)$ and $p'_1, p'_2, p'_3 \in S'$. Then there exists $p_1, p_2, p_3 \in S$ with $f(p_1) = p'_1, f(p_2) = p'_2, f(p_3) = p'_3$

$$\begin{aligned} \alpha_{f(\mathcal{P})}(p'_1 p'_2 p'_3) \wedge \alpha_{f(\mathcal{P})}(p'_2) &\leq \alpha_{\mathcal{P}}(p_1 p_2 p_3) \wedge \alpha_{\mathcal{P}}(p_2) \\ &= 0 \end{aligned}$$

On the other hand,

$$\begin{aligned} \alpha_{f(\mathcal{P})}(p'_1 p'_2 p'_3) + \beta_{f(\mathcal{P})}(p'_1 p'_2 p'_3) \wedge \alpha_{f(\mathcal{P})}(p'_2) + \beta_{f(\mathcal{P})}(p'_2) \\ = \alpha_{\mathcal{P}}(p_1 p_2 p_3) + \beta_{\mathcal{P}}(p_1 p_2 p_3) \wedge \alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2) \\ = 0 \end{aligned}$$

Similarly, we can prove that $\gamma_{f(\mathcal{P})}(p'_1 p'_2 p'_3) \vee \gamma_{f(\mathcal{P})}(p'_2) = 1$. Hence, $f(\mathcal{P}) \in PFIAI(S')$. $\square$

Theorem 3.6. *Let $f : S \rightarrow S'$ be a semigroup homomorphism, and let $\mathcal{R} \in PFS(S')$. If $\mathcal{R} \in PFIAI(S')$, then $f^{-1}(\mathcal{R}) \in PFIAI(S)$.*

Proof. Let $\mathcal{R} \in PFIAI(S')$ and $p_1, p_2, p_3 \in S$.

$$\begin{aligned} \alpha_{f^{-1}(\mathcal{R})}(p_1 p_2 p_3) \wedge \alpha_{f^{-1}(\mathcal{R})}(p_2) &= \alpha_{\mathcal{R}}(f(p_1 p_2 p_3)) \wedge \alpha_{\mathcal{R}}(f(p_2)), \\ &= \alpha_{\mathcal{R}}(f(p_1) f(p_2) f(p_3)) \wedge \alpha_{\mathcal{R}}(f(p_2)), \\ &= 0. \end{aligned}$$

Also,

$$\begin{aligned} (\alpha_{f^{-1}(\mathcal{R})}(p_1 p_2 p_3) + \beta_{f^{-1}(\mathcal{R})}(p_1 p_2 p_3)) \wedge (\alpha_{f^{-1}(\mathcal{R})}(p_2) + \beta_{f^{-1}(\mathcal{R})}(p_2)) \\ = (\alpha_{\mathcal{R}}(f(p_1) f(p_2) f(p_3)) + \beta_{\mathcal{R}}(f(p_1) f(p_2) f(p_3))) \wedge (\alpha_{\mathcal{R}}(f(p_2)) + \beta_{\mathcal{R}}(f(p_2))) \\ = 0. \end{aligned}$$

Similarly, it can be seen that $\gamma_{f^{-1}(\mathcal{R})}(p_1 p_2 p_3) \vee \gamma_{f^{-1}(\mathcal{R})}(p_2) = 1$. Thus, $f^{-1}(\mathcal{R}) \in PFIAI(S)$. $\square$

Theorem 3.7. *Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S_1)$ and $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(S_2)$. If $\mathcal{P} \in PFIAI(S_1)$ and $\mathcal{R} \in PFIAI(S_2)$, then $\mathcal{P} \times \mathcal{R} \in PFIAI(S_1 \times S_2)$.*

Proof. Put $u_{\mathcal{P}}(p) = \alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)$ and $u_{\mathcal{R}}(r) = \alpha_{\mathcal{R}}(r) + \beta_{\mathcal{R}}(r)$. By the definition of the Cartesian product,

$$\alpha_{\mathcal{P} \times \mathcal{R}}(p, r) + \beta_{\mathcal{P} \times \mathcal{R}}(p, r) = u_{\mathcal{P}}(p) \wedge u_{\mathcal{R}}(r).$$

For $(p_i, r_i) \in S_1 \times S_2$, $i = 1, 2, 3$, we have

$$\begin{aligned} & \alpha_{\mathcal{P} \times \mathcal{R}}((p_1, r_1)(p_2, r_2)(p_3, r_3)) \wedge \alpha_{\mathcal{P} \times \mathcal{R}}(p_2, r_2) \\ & \leq \alpha_{\mathcal{P}}(p_1 p_2 p_3) \wedge \alpha_{\mathcal{P}}(p_2) = 0. \end{aligned}$$

Similarly,

$$\begin{aligned} & (\alpha_{\mathcal{P} \times \mathcal{R}}((p_1, r_1)(p_2, r_2)(p_3, r_3)) + \beta_{\mathcal{P} \times \mathcal{R}}((p_1, r_1)(p_2, r_2)(p_3, r_3))) \\ & \wedge (\alpha_{\mathcal{P} \times \mathcal{R}}(p_2, r_2) + \beta_{\mathcal{P} \times \mathcal{R}}(p_2, r_2)) \\ & \leq u_{\mathcal{P}}(p_1 p_2 p_3) \wedge u_{\mathcal{P}}(p_2) = 0. \end{aligned}$$

Finally,

$$\begin{aligned} & \gamma_{\mathcal{P} \times \mathcal{R}}((p_1, r_1)(p_2, r_2)(p_3, r_3)) \vee \gamma_{\mathcal{P} \times \mathcal{R}}(p_2, r_2) \\ & = \gamma_{\mathcal{P}}(p_1 p_2 p_3) \vee \gamma_{\mathcal{P}}(p_2) \vee \gamma_{\mathcal{R}}(r_1 r_2 r_3) \vee \gamma_{\mathcal{R}}(r_2) = 1. \end{aligned}$$

Therefore, $\mathcal{P} \times \mathcal{R} \in PFIAI(S_1 \times S_2)$. □

4. CONCLUSION

In this study, picture fuzzy interior antiideals in semigroups were introduced and investigated within the type-3 framework for picture fuzzy algebraic operations established in [9]. This framework provides a complete-lattice setting for these operations, addressing a limitation of the type-1 order. The results establish a correspondence between classical interior antiideals and their picture fuzzy characteristic functions, and characterize picture fuzzy interior antiideals by means of their nonempty level subsets. They also show that this class is closed under nonempty intersections and is preserved under isomorphic images, inverse images under semigroup homomorphisms, and Cartesian products, whereas unions need not preserve the picture fuzzy interior antiideal property. These findings extend the type-3 algebraic study of picture fuzzy sets to interior antiideals and provide a basis for investigating related notions in other algebraic structures, such as rings, near-rings, and ordered semigroups.

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The Declaration of Conflict of Interest/ Common Interest

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The Declaration of Ethics Committee Approval

This study does not require ethical committee permission or any special permission.

The Declaration of Research and Publication Ethics

The author declared that they comply with the scientific, ethical, and citation rules of Journal of Universal Mathematics in all processes of the study, and that they do not make any falsification of the data collected. Besides, the author declared that Journal of Universal Mathematics and its editorial board have no responsibility for any ethical violations that may be encountered, and this study has not been evaluated in any academic publication environment other than Journal of Universal Mathematics.

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