EGE AKADEMİK BAKIŞ / *EGE ACADEMIC REVIEW*

Cilt 26 • Sayı 1 • Ocak 2026
SS. 43/62
Doi: 10.21121/eab.20260104
Başvuru Tarihi: 04.03.2025 • Kabul Tarihi: 28.09.2025

Article Type: *Research Article*

Forecasting The Volatility of Bist 100 Index Return with Linear and Nonlinear Time Series Models

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ABSTRACT

The successful modeling and forecasting of volatility, which is the most important element of risk indicators, minimizes financial uncertainties. Classical volatility models are insufficient to forecast structural changes in economic variables. Hybrid models that integrate the benefits of several model architectures have become more significant as the amount of neural network-based research has increased recently. The purpose of the research is to show that mixed models are more accurate and consistent when it comes to predicting variable volatility. For this purpose, the return volatility of the Borsa Istanbul 100 index was modeled, and forecasting performance results were compared with hybrid models. According to the findings, the best forecasting performance was achieved with hybrid structures containing the exponential GARCH-Artificial Neural Networks (MSEGARCH-ANN) combination. It can be said that hybrid models are superior in the risk analysis of volatile financial instruments and in the estimation of macroeconomic variables in general.

Keywords: Volatility, Artificial Neural Networks with Hybrid Models, BIST 100 Index Return.

JEL Classification Codes: C22, C45, C53, G17

Referencing Style: APA 7

INTRODUCTION

Financial market instruments are immediately affected by various events, including political and economic changes. These markets are particularly exposed to cyclical risks arising from unpredictable or theoretically unexplained factors. While diversification is commonly employed to mitigate risks, it is insufficient to protect against all types of uncertainty. Since the level of risk in financial instruments can vary over time, monitoring risk characteristics is crucial for minimizing ambiguity and effectively managing investment risks.

The risk arising from the variability of financial instrument returns refers to the probability that a stock's actual profit may deviate from its expected return. The central concept in risk management is volatility, which represents the uncertainty associated with the returns of an asset or financial instrument (Hull, 2006: 758). In financial time series, volatility also defined as the measure of changes occurring in financial markets over a specific period can manifest in both the short term, over a few hours, and the long term, over periods of 15–20 years. While economic and political developments tend to generate low levels of volatility, financial events in the market can lead to increased volatility. Indeed, macroeconomic data that indicate low levels are associated with low volatility, whereas high-level data are associated with high volatility (Sevüktekin and Çınar, 2006: 244). Therefore, market developments and investors' potential exposure to significant losses underscore the critical importance of understanding volatility.

Traditional econometric models assume that the lagged values of the error term are homoscedastic. However, in modern financial markets, where variables often exhibit multidirectional relationships, this assumption frequently does not hold, resulting in heteroskedastic structures. Predicting models with heteroskedasticity using conventional linear time series approaches, such as ARIMA, is therefore insufficient. To account for market fluctuations, researchers commonly employ autoregressive conditional heteroskedasticity (ARCH) models, which are parametric methods developed by Engle (1982). The nonlinear and symmetric ARCH model often requires a high number of lags, necessitating numerous parameters for accurate

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This study is derived from Hakan ÖNDES's doctoral thesis titled "Application of Linear and Nonlinear Time Series Analysis on Some Selected Macroeconomic Variables".

prediction. To address this limitation, Bollerslev (1986) introduced the generalized autoregressive conditional heteroskedasticity (GARCH) model (Işığıçok, 1999: 2).

Furthermore, in the context of modeling the conditional volatility of shocks, Taylor (1986) developed the Absolute Value GARCH (AVGARCH) model. Nelson (1990) argued that the asymmetric response of volatility in financial instruments traded on developing markets to incoming information renders standard GARCH models inadequate. Volatility models that account for asymmetric effects demonstrate that negative shocks tend to have larger and different impacts compared to positive shocks (Özden, 2008: 345). To address these asymmetries, Zakoian (1994) introduced the Threshold GARCH (TGARCH) model by incorporating a leverage effect into the conditional heteroskedasticity equation. Nelson (1991) developed the Exponential GARCH (EGARCH) model, while Ding, Granger, and Engle (1993) proposed the Asymmetric Power ARCH (APARCH) model. Despite the development of these alternative models, their predictive performance in forecasting market fluctuations often remains limited.

According to Lamoureux and Lastrapes (1990), using a volatility model that does not allow for structural changes in its forecasted parameters is insufficient for reliable estimation. Therefore, the ARCH models discussed above require specification within a different structural framework. Additionally, traditional ARCH models are not responsive to low and high volatility fluctuations, which are important for understanding market dynamics. Financial markets often experience periods of contraction and expansion that vary in effect and duration. To address these stochastic changes in volatility, GARCH models based on the Markov regime-switching approach (MS-GARCH) have been developed. The first combination of MS-GARCH with ARCH models occurred in 1989 through the Switching ARCH (SWARCH) model applied by Hamilton to return series. For financial time series, MS-GARCH has been utilized by Bildirici and Ersin (2014), Kula and Baykut (2017), and Tan et al. (2021). In the context of Borsa Istanbul index returns, Çavdar and Aydın (2017), Kula and Baykut (2017), Kutlu and Karakaya (2019), and Kaya and Yarbaşı (2021) have applied SWARCH and MS-GARCH models. Similarly, studies on financial markets in other countries, such as Marcucci (2005), Hu and Shin (2008), Augustyniak (2014), Abounoori et al. (2016), and Korkpoe and Howard (2019), have also employed the MS-GARCH approach.

In recent years, researchers have increasingly employed non-parametric models, such as artificial neural networks and fuzzy logic, to analyze prices or returns in financial time series that are difficult to forecast using parametric models. Beyond these approaches, more advanced techniques, known as hybrid (or mixed) models, have been developed to improve the reliability of time series modeling and forecasting. Studies have shown that hybrid models often outperform both traditional parametric and non-parametric models in forecasting accuracy (Güreşen and Kayakutlu, 2008; Bildirici and Ersin, 2014; Lahmiri and Boukadoum, 2015). Hybrid models benefit from a structure that simultaneously leverages the strengths of both parametric and non-parametric approaches.

In the context of BIST 100 Index returns, existing research on hybrid models is not only scarce but also methodologically limited, as prior studies generally focus on single-layer combinations such as GARCH–ANN. This study addresses this gap in the literature by proposing a multi-layered hybrid forecasting framework that simultaneously incorporates conventional time series models (ARMA), volatility models (ARCH, GARCH, TGARCH), regime-switching variance models (Markov Regime Switching-GARCH, EGARCH, GJRGARCH), and artificial neural networks (ANN). The principal novelty lies in employing ANN to capture regime-dependent volatility dynamics—an aspect that has rarely been explored in previous research. By combining the statistical rigor of econometric modeling with the adaptive learning capacity of machine learning, this framework reveals both linear and non-linear dynamics under a unified structure. In doing so, the study goes beyond testing individual models' predictive performance and demonstrates the methodological synergy achieved through their integration, thereby offering a "beyond hybrid" perspective to the financial time series forecasting literature. Unlike earlier works that rely on two-dimensional hybrids, this study demonstrates the added value of a layered integration that simultaneously captures regime shifts and non-linear dynamics.

In the study, first of all, studies in the literature that have been carried out on the return volatility of index series using the MS-GARCH structure and hybrid models were reviewed, and the results are presented in the Literature Review section. The third part of the paper addresses the structures of the datasets and the theoretical framework of the models that were used. The fourth part presents the modeling of the volatility of BIST 100 Index returns, the forecasting process of this volatility, and the comparison of the models based on three different forecasting performance criteria. Finally, in the last part, the results are discussed, and recommendations that are seen fit are made.

LITERATURE REVIEW

Stock markets show constant fluctuations under the influence of several positive and negative shocks experienced in countries. When these markets are substantially affected by these positive or negative shocks, in turn, their risk-return performance is significantly affected. The prevalence of financial crises or major financial collapses has boosted the appeal of models that take into account hybrid structures based on regime switching, which have remained sensitive to recurrent market situations like contraction and growth. Therefore, hybrid models based on regime switching are appropriate techniques to capture structural shifts in the world of finance and main developments in stock market dynamics.

In the relevant literature, there are several studies that have used ARCH approaches, ANNs, and Markov regime switching approaches. As a different study in terms of its approach, this study also discussed previous studies where hybrid models based on Markov regime switching have been used.

In financial time series, the first scholars to utilize models based on Markov regime flipping were Cai (1994), Hamilton, and Susmel (1994). When the tardy values of conditioned variances are removed from the variance equation, the probability function may be quantitatively calculated. The prediction procedure becomes unfeasible when a GARCH-type model is employed since the study of the probability in a Markov chain with K regimes necessitates an integration of all possible K*T (T: time period) routes. This issue has been resolved by these researchers by removing the influence of conditional variances particular to a regime. Gray (1996) asserts that the conditional spectrum of yields varies regardless of the paradigm route and that the GARCH formula combines the conditional projection of prior variation with the invisible path of the regimes.

The use of MS-GARCH-type structures in financial markets has expanded as a result of these advancements. From January 1998 to October 2003, Marcucci (2005) studied how well MS-GARCH systems and other scenario structures simulated the return volatility of the S&P 100 Index. It was observed that the MS-GARCH structures provided more successful results in comparison to the standard GARCH structures. Using the stock market indices of developing countries in East Asia, Hu and Shin (2008) carried out MS-GARCH modeling.

Bildirici and Ersin (2014) combined the MS-GARCH structure and its derivatives with ANNs and used them to model daily stock returns on the BIST 100 Index. To test forecasting performance, they used the MAE, MSE, and RMSE standard, as well as Diebold-Mariano predictive reliability evaluations. Their outcomes showed that the MS-GARCH model of Gray (1996) is more promising than its fractionally integrated and asymmetric power variants, and the best forecasting results are obtained with models based on ANNs. Augustyniak (2014) developed a new technique for calculating the maximum probability estimator with equilibrium variance-covariance matrices of the MS-GARCH model that is based on the Monte Carlo Expectation-Maximization technique and significance sampling. The efficiency of the suggested method was demonstrated through simulations and empirical trials, and its practical implementation was examined.

Abounoori et al. (2016) analyzed some GARCH models based on their capacity to predict Tehran Stock Exchange (TSE) fluctuations. Regarding the identification and forecasting of volatility in terms from 1-day to 22-day periods, their analyses included GARCH equations using fat-tailed remnant restricted with Gaussian distributions. The results showed that the AR(2) MS-GARCH-ged (Generalized Error Distribution) model was more successful than the other models in the 1-day forecasting term. Additionally, while the AR(2)-MS-GARCH-ged and AR(2)-MS-GARCH-t (t-Distribution) models had a more consistent performance than the other models in the 5-day forecasting term, the model with the best performance in the 10-day forecasting term was AR(2)-MS-GARCH.

Kula and Baykut (2017) sought to use the closing prices on a daily basis of the index throughout the period in order to ascertain the fluctuation structure of the BIST 100 Banks Index between 2 January 1996 and 31 December 2016. With the MS-GARCH models they used, they determined that the BIST 100 Banks Index had low risk-regime persistence, and the index did not show consistency in its transition to a low-risk regime when it was in a high-risk regime. The BIST 100 Banks Index was shown to have a significant degree of volatility persistence in both regimes.

Çavdar and Aydın (2017) used the GARCH and SWARCH structures to examine the fluctuation of the BIST Corporate Governance Index. Their findings showed that the regime-switching-based SWARCH model outperformed traditional GARCH models in assessing the fluctuation of the BIST Corporate Governance Index.

Based on the assumption that financial markets are influenced by underlying economic developments, Korkpoe and Howard (2019) conducted a detailed study including Botswana, Ghana, Kenya, and Nigeria to predict the stock return risk in stock markets in Sub-Saharan Africa. They detected heterogeneity in the volatility structure of these markets and demonstrated that the Akaike information criteria of the 2-regime MS-GARCH models better identify the heteroskedastic return-generation processes in these markets. They came to the conclusion that MS-GARCH structures were the best models among those chosen to forecast the future volatility of returns in the financial sectors being studied.

Kutlu and Karakaya (2019) applied a two-regime MS-GARCH approach to analyze the BIST Tourism Index's unpredictability for the period between May 2003 and September 2018. In their analyses, which revealed the differences among three periods as before, during, and after the 2008 crisis, they determined that the volatility of the index could not return to its pre-crisis structure, the volatility continued, therefore there was more volatility in the after the crisis era than in prior to the crisis.

Kaya and Yarbaşı (2021) modeled volatilities occurring in the BIST 100 Index. Considering the period from April 1993 to April 2018, they used an MS-GARCH model for situations of shattered norm, and extreme variability. The system parameters derived for the gauge were important according to their calculations, which included the three-regime MS-GARCH model and the modeled regimes were valid for the index.

In order to simulate the volatility interactions of a Bitcoin (BTC) return sequence, Tan et al. (2021) proposed GARCH models based on Markov regime-switching approach including time-sensitive (TV) likelihoods of transition (TV-MS-GARCH). They did this by using daily searching on Google and regularly Bitcoin (BTC) transactions as external factors, both separately and together. They conducted thorough assessments with similar models, comprising GARCH, GJRGARCH, threshold GARCH, consistent shift probability MS-GARCH, and MS-GJRGARCH, so as to determine the simulation performance of the proposed models. They showed that the TV-MS-GARCH structures with imbalanced and fat-tailed dispersion match the data well compared to other structures based on the Akaike definition of information and other baseline criteria.

Financial time series forecasting is becoming more and more crucial as financial markets adjust to the quickly shifting climate, according to He et al. (2022). A novel deep learning ensemble model that combines CNN, LSTM, and ARMA models is suggested in this context. Simultaneous capture of linear and nonlinear data characteristics is the model's goal. The findings of empirical study show that, in comparison to individual models, the suggested model provides better predicting accuracy and resilience.

According to Kontopoluo et al. (2023), as artificial intelligence capabilities progress, machine and deep learning approaches are gradually replacing ARIMA models, which have long been popular in time series forecasting. By integrating the advantages of both techniques, hybrid models offer better forecasting accuracy, according to their study, which contrasts the use of ARIMA and AI-based models in various industries (financial, healthcare, weather, etc.).

Accurate financial time series forecasting is essential for risk management and investment choices, according to Cappello et al. (2025). Artificial intelligence techniques like ANN are becoming more and more popular as a result of traditional approaches' inability to adequately capture market dynamics. This work presented and evaluated a new hybrid model that combines ARIMA with ANN-based models like LSTM and GRU using data on stocks, exchange rates, and Bitcoin prices. According to the findings, the hybrid model performed better than both conventional and earlier hybrid models, and the Diebold-Mariano test indicated that these differences were statistically significant.

For financial time series forecasting, Agarwal et al. (2025) suggest a new hybrid model based on SVM and LSTM and emphasize the significance of stock market swings as economic indicators. While LSTM improves prediction accuracy by accounting for the impact of prior data, the SVM technique separates non-stationary data and makes it measurable. When tested using a variety of financial data, including the HSI, SENSEX, S&P500, and WTI, the model outperformed both individual and pre-existing hybrid models in terms of predicting.

In terms of its methodological breadth and level of integration, this study is very different from previous techniques found in the literature. The shared research often create an ensemble model that combines machine learning techniques with classical time series models, or they blend classical time series models with artificial neural networks. A hybrid architecture of machine and deep learning techniques is also used in certain investigations. On the other hand, this study concurrently incorporates volatility models from the ARCH-GARCH family (GARCH, TGARCH, EGARCH, and GJRGARCH), classical time series

models (ARMA), and their extended versions with Markov regime switching. Additionally, linear, nonlinear, and regime-dependent dynamics are all simultaneously represented by integrating artificial neural networks into this multilayered framework. In this regard, the suggested method outperforms current models in the literature by providing both a hybrid and a multilayer integrative modeling technique. The results demonstrate that the interaction between model components significantly influences the forecasting performance, in contrast to the single or limited combination models used in earlier research. In this regard, the findings provide a new methodological framework that can be incorporated and expanded upon in the body of existing literature.

METHODOLGY

In this study, ARCH models, Markov regime-switching-based autoregressive conditional heteroskedasticity models, synthetic brain network simulations, and mixed approaches as the methodology. Brief information on these models is provided below.

ARCH Models

In 1982, Engle introduced the ARCH framework to model variations in conditions when the stochastic process changes over time while the dependent deviations remain constant. He represented these variations using a qth-order autoregressive process, which became known as the qth-order ARCH model. Engle (1982) specified the first-order autoregressive process, AR(1), as the main equation as follows:

$$Y_t = \phi Y_{t-1} + \varepsilon_t \tag{1}$$

Here, ε_t is a mistake element that has a fixed variance and zero indicate, while the absolute average of Y_t is zero, and its conditional mean is ϕY_{t-1}.

The standard approach to heteroskedasticity involves the inclusion of a lagged value of an exogenous variable X_t estimating the variance. This zero-mean model is as follows:

$$Y_t = \varepsilon_t X_{t-1} \tag{2}$$

When the consecutive values of the exogenous variable X_t here are constant, the series Y_t has a white-noise technique with stable variance. Moreover, a model including the reliant variation is listed below:

$$Y_t = \varepsilon_t Y_{t-1} \tag{3}$$

The conditional variance of Equation 3 is $\sigma^2 Y_{t-1}$. In this case, the unilateral variation is either null or eternity. As a model more optimal for this case, the following ARCH model is proposed:

$$Y_t = \varepsilon_t h_t^{1/2} \tag{4}$$

$$h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 \tag{5}$$

The variance of ε_t in this model is equal to 1, and the autoregressive conditional heteroskedasticity (ARCH) theory is the name given to the concept. When the information set is ψ_t, the assumption of normality is adapted to the model in Equation 4, and after the zero mean is replaced with $X_t\beta$ as a mean model, the following ARCH(p) regression model is obtained:

$$\begin{aligned} &Y_t|\psi_{t-1} \sim N(X_t\beta, h_t) \\ &h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2 \\ &\varepsilon_t = Y_t - X_t\beta \end{aligned} \tag{6}$$

Here, p refers to to the ARCH method's sequence, α is the scalar with undefined characteristics. (Engle, 1982:756; Işığıçok, 1999:3). Indeed, for $p = 1$, the model becomes ARCH(1).

The ARCH concept, however, has several limitations. One key restriction is that the conditional variance (h_t) must always be positive. To satisfy this requirement, the α_i parameters must meet the condition of positive and finite variance. Additionally, each α_i parameter and their total sum must be less than one (Işığıçok, 1999: 4). Ensuring that the sum of the parameters is less than one guarantees finite variance for the model (Greene, 1993: 146).

Bollerslev (1986) created the GARCH(p,q) structure via expanding the ARCH(p) model to include q delayed values of the conditional variance:

$$\begin{aligned} &Y_t|\psi_{t-1} \sim N(X_t\beta, h_t) \\ &h_t = \alpha_0 + \sum \alpha_i\varepsilon_{t-i}^2 + \sum \beta_j h_{t-j} \\ &\varepsilon_t = Y_t - X_t\beta \end{aligned} \tag{7}$$

Here, when α_0 the constant term, and $i = 1,2,\ldots,p$, α_i, indicate the ARCH parameters, and when $j = 1,2,\ldots,q$, β_j, indicate the GARCH parameters. In the GARCH(p,q) structure, the p and q lag lengths can be determined by using the AIC and SCI criteria.

However, like the ARCH approach, the GARCH structure further has some limitations. The parameters relevant to the GARCH system's contingent variation (α_i and β_j) must meet the positive and finite variance condition. Certainly, as in the ARCH model, this condition also meets the finite variance inference for the model.

The GARCH model cannot capture negative or positive asymmetries because it assumes that the error terms follow a symmetric distribution. Another limitation of the GARCH model is its inability to account for the persistence of shocks in the conditional variance. To overcome these issues, Nelson (1991) developed the Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model. This model addresses the main drawbacks of the standard GARCH approach. The EGARCH model is expressed as follows (Nelson, 1991: 350):

$$\log \sigma_t^2 = \alpha_t + \sum_{k=1}^{q} \beta_k g(z_{t-k}) \quad (8)$$

In Equation 8, $z_t = \varepsilon_t / \sqrt{h_t}$ express the normalized errors. The function g(.) in the model is in the form of:

$$g(z_t) = \theta z_t + \gamma\{|z_t| - E(|z_t|)\} \quad (9)$$

Here, θz_t refer to positive or negative shocks. The function $\gamma\{|z_t| - E(|z_t|)\}$ includes the value of ε_t in the structure. The EGARCH approach accounts for the direction of irresolution.

The generalized expression of the EGARCH model is as follows:

$$logh_t = \alpha_0 + \beta logh_{t-1} + \delta_1 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \delta_2 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \quad (10)$$

While the δ_1 and δ_2 components in the structure represent the tendency of irresolution, the β component represents the persistence of shocks. h_t varies depending on the quantity and sign of the lagged errors. When $\delta_2 \neq 0$, the model will be asymmetric (Altındiş, 2005:35).

Additionally, the Threshold GARCH (TGARCH) approach is produced via supplementing a leverage (impulse) fickle to the conditional variance expression (Zakoian, 1994:936). The TGARCH model's conditional variance equation looks like this:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \beta h_{t-1} \quad (11)$$

The γ parameter in the equation refers to the leverage effect. Due to the inclusion of the pry impact in the approach, the dummy variable d_{t-1} is added, and the value of d_{t-1} is 0 for $\varepsilon_{t-1} \geq 0$ and 1 for $\varepsilon_{t-1} < 0$. Clearly, the significant and positive values of the γ parameter show the inclusion of the pry impact (Işığıçok, 1999:7).

In the TGARCH model, an unexpected rise in the series is interpreted as a positive development, and the α_0 parameter influences the conditional variance; conversely, an unexpected fall is interpreted as a negative development, and the α_0 and γ parameters influence the conditional variance. When negative shocks occur, financial market volatility increases to a greater extent. The TGARCH model is used to explain this scenario, which is viewed as a leverage impact on returns. It should be emphasized that the TGARCH model becomes the GJRGARCH model when the variance is substituted for the standard deviation in the conditional variance expression. Here, GJR is made up of the initials of the model's creators, Glosten, Jaganathan, and Runkle (1993)..

Furthermore, Taylor (1986) and Schwert (1990) suggested the Absolute Value GARCH (AVGARCH) structure. The fundemantal formulation of the structure appears to be shown below: η_{11} and η_{21} are the martingale difference coefficients (that is, for $j \geq 1$, $E(\eta_t) = 0$, and $\mathrm{cov}(\eta_t, \eta_{t-j}) = 0$). Nevertheless, $\{\eta_t\}$ is usually independent and non-homogeneously distributed.

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i h_{t-i} (|\varepsilon_{t-i} - \eta_{21}| + \eta_{11}(\varepsilon_{t-i} - \eta_{21})) + \sum_{j=1}^{p} \beta_j h_{t-j} \quad (12)$$

Here, there are the constraints $\alpha_0 > 0, \alpha_i > 0, \beta_j > 0$, and $(\alpha_i + \beta_j) < 1$. In this structure, the error term of this conditioned variation is explained by the absolute value of the past period considering the old qualities of the conditional variance.

The APGARCH structure that was developed by Ding, Granger, and Engle (1993) is as follows:

$$h_t^{\delta} = \alpha_0 + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{t-i}| + \gamma \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{p} \beta_j h_{t-j}^{\delta} \quad (13)$$

δ is the power term, and γ values are the asymmetry parameters. The equation includes the constraints $\alpha_0 > 0, \alpha_i \geq 0, \beta_j > 0, \delta > 0$ and $-1 < \gamma < 1$. Accordingly, when $\beta_j = 0$ in this model, the model becomes the Asymmetric Power ARCH (APARCH) model.

Markov Regime-Switching-Based Autoregressive Conditional Heteroskedasticity Models

The Markov regime switching approach that was proposed by Hamilton (1989) has a process in the form of $P\{s_k = j \mid s_{k-1} = i, = k, \ldots\} = P\{s_k = j \mid s_{k-1} = i\} = p_{ij}$, where i is the 1st regime, j is the 2nd regime, k is the number of regimes, is the state variable that indicates the changes in the regimes, and s_k is the changeover likelihood that the ith period to the jth period. In this equation, which expresses the two-regime Markov chain, regime changes are able to be evaluated based on several periods depending on the state variable s_k. When $i \neq j$ in the equation above, $p_{ij} = 1 - p_{ii}$. At this stage, the following is an expression for a two-regime Markov procedure with a changeover likelihood dependent on the status element sk:

$P_r[s = 0 \mid s = 0\,] = p$: The likelihood of remaining in the low regime while in it
$P_r[s = 1 \mid s = 0\,] = 1 - p$: The likelihood that the low regime will give way to the high regime
$P_r[s = 1 \mid s = 1\,] = q$: The likelihood of remaining in the high regime while in it
$P_r[s = 0 \mid s = 1\,] = 1 - q$: The likelihood that the high regime will give way to the low regime

As there are two regimes here, the value k in the state variable s_k becomes 2. Low volatility is expressed as $i = 0$, and high volatility with abrupt spikes is expressed as $j = 1$.

The contingent mean and dispersion, system procedure and contingent variation are all components of a typical autoregressive conditional heteroskedasticity model based on Markov regime-switching. The following is the definition of the conditional mean that displays a random walk process:

$$v_t = \mu_t^{(i)} + \varepsilon_t 04\ Math\ 11_06 = \delta^{(j)} + \varepsilon_t \tag{14}$$

In the equation above, regime changes are expressed as i $(i=1,2)$. The process with a variance of 1 and a mean of 0 is included in the variable n_t. The conditional variance equations for the MS-GARCH, MS-EGARCH, and MS-GJRGARCH procedures, three of the Markov regime-switching-based GARCH models employed in this investigation, are as follows:

$$h_{t_{s_t}} = \alpha_{0_{s_t}} + \alpha_{i_{s_t}} \varepsilon_{t-i}^2 + \beta_{j_{s_t}} h_{t-j} \tag{15}$$

$$logh_t = \alpha_{0_{s_t}} + \beta_{s_t} logh_{t-i} + \delta_{1_{s_t}} \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \delta_{2_{s_t}} \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \tag{16}$$

$$h_t^2 = \alpha_{0_{s_t}} + \alpha_{1_{s_t}} \varepsilon_{t-i}^2 + \gamma_{s_t} \varepsilon_{t-i}^2 d_{t-i} + \beta_{s_t} h_{t-i}^2 \tag{17}$$

For instance, the MS-GARCH(1,1) contingent variation is expressed as follows:

$$h_{t_{s_t}} = \alpha_{0_{s_t}} + \alpha_{1_{s_t}} \varepsilon_{t-1}^2 + \beta_{1_{s_t}} h_{t-1} \tag{18}$$

The other models can also be expressed similarly. The model given in Equation 18 was expanded by Gray (1996) and Klaassen (2002) to include low and high volatility values as follows:

$$h_{t_{s_t}} = \left[\alpha_{0_{s_t}} + \alpha_{1_{s_t}} \varepsilon_{t-1}^2 + \beta_{1_{s_t}} h_{t-1}\right] \mid [s_t = 0] + \left[\alpha_0 + \alpha_{1(s_t)} \varepsilon_{t-1}^2 + \beta_{1(s_t)} h_{t-1}\right] \mid [s_t = 1] \tag{19}$$

While regime switches in the equation are shown with $s_t, s = 0$, refers to the deep fluctuation regime, and $s = 1$ describes to the huge fluctuation.

Artificial Neural Network Models

A structure that resembles neurons and neural connections that are the most fundamental elements of the human brain and nervous system in form and function and operates in the form of a mathematical equation of biological neural structures is called an artificial neural network (ANN). ANNs, which allow the simulation of neural cells in the human brain, are algorithms that can produce new data based on previous data.

Neurophysiologist Warren Mc. Culloch and mathematician Walter Pitts (1943) produced a rudimentary neural network using electrical circuits. Since their research, significant steps in ANNs have been taken with the advancement of computer technology, especially in recent years.

In artificial neural networks (ANNs), data are typically divided into two parts: training data and test data. The training data are used to learn the underlying relationship structure (model) among the variables under study, while the test data are employed to generate predictions based on the learned model. This approach makes ANNs highly suitable for solving problems that do not conform to any predefined model pattern. Furthermore, because volatility forecasting or modeling in time series is not strictly theory-dependent, ANNs have become one of the most widely used techniques in this area.

The general structure of an artificial neural network (ANN) consists of three layers: the input layer, the hidden layer, and the output layer. The input layer receives information from external sources, and the number of independent variables in the input layer corresponds to the number of neurons it contains. The hidden layer processes the information received from the input layer and can consist of one or more sublayers. The researcher determines the number of sublayers and the number of neurons in each sublayer through a trial-and-error process. These trials aim to identify the configuration that provides optimal performance.

Information in the hidden layer is processed through transfer (activation) functions within the neurons. The activation function, which is chosen by the user, is a key factor affecting the performance of the network. The hidden layer provides the ANN with the capability

to model non-linear relationships. The output layer processes the information from the hidden layer and delivers the results to the user. Data obtained through the connections in the hidden layer are transmitted to the external environment as the network's direct output. The number of variables in the output layer corresponds to the number of dependent variables used in the analysis.

There are two reasons why ANNs are prevalently utilized in the fields of economics and finance today. First, ANNs do not have linearity-related constraints for the parameters to be predicted. Second, ANN models do not have any presumptions concerning the distribution in the time series to which they will be applied.

Hybrid Models

In this study, two different hybrid structures were used. The first structure included models consisting of combinations of ARCH models and ANN models. In this system that was proposed by Roh (2007), first of all, the time series that is studied is predicted using ARCH models. Then, the theoretical values of this predicting model are used as input in the ANN. In this method, it is aimed to forecast volatility better by making the learning process of the ANN algorithm easier.

The second structure included models consisting of combinations of Markov regime-switching-based autoregressive conditional heteroskedasticity models and ANN models developed by Bildirici and Ersin (2014). In this structure, like the other structure, the Markov regime-switching-based autoregressive heteroskedasticity models are predicted first. Next, the theoretical values of this forecasting model are used as input in the ANN. This structure is expected to possess superior predicting abilities than the hybrid structures described above under the impact from the Markov regime switching structure in predicting volatility.

FINDINGS and DISCUSSION

In this study, the BIST 100 Index series included data on the closing prices in the period between January 1997 and June 2025. For the test data to be used in the ANN, the forecasting data for the January 2025–June 2025 time frame were used. For the stationarity of the series, a return series was created via calculating the index series' logarithmic discrepancy.

$$r_t = 100[ln(P_t) - ln(P_{t-1})] \quad (20)$$

Here, r_t represents returns, and P_t represents closing prices for the BIST 100 Index series at time t. The time-path plot of the obtained return values is shown in Figure-1.

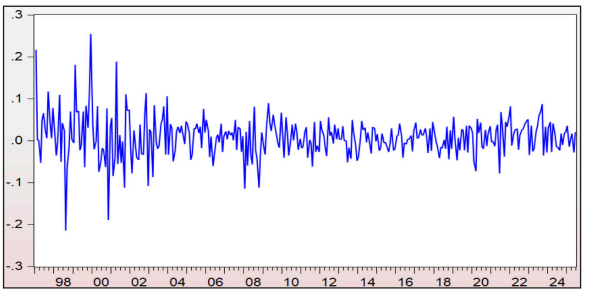

Figure 1: Return Series

Source: Created by the authors in the Eviews program.

As seen in Figure-1, the return series had a trend around the zero mean. This showed that the mean return was zero as expected, and there was no systematic return. Traditional and structural break unit root tests were conducted using EViews 12 to evalute the stagnant of the series in question, and the results shown in Table-1 were obtained.

The ADF, PP, and KPSS tests at level, as well as the findings displayed in Table 1, showed that the return series was stationary and devoid of unit roots. Following this, the return series was put through the Carrion-i-Silvestre unit root test with numerous structural breaks in Gauss 6 and the Lee-Strazicich structural break unit root test in RATS 8. The outcomes of these tests are displayed in Tables 2 and 3, respectively.

According to the structural break unit root tests on the return series whose results are shown in Table-2 and Table-3, the series was stationary at level for both tests. Before the ARCH model results for the return series, the series was forecasted using conventional Box-Jenkins methods in EViews 12, and the most significant model structures are shown in Table-4.

As seen in the results demonstrated in Table-4, between the three models, according to the information criteria and other statistical criteria, the ARMA(3,3) model was determined as the best model. As seen at the bottom of Table-4, at the stage following model prediction, using the residuals of the ARMA(3,3) model above, the presence of conditional heteroskedasticity was demonstrated with the ARCH LM test. Thus, alternative autoregressive conditional heteroskedasticity model predictions were carried out, and the results of the models showing the best performance are presented in Table-5.

According to the ARCH model predictions that were carried out, considering the conditions of in the variance model, the statistically significant coefficients, and the information criteria, the optimal model was found as ARMA(3,3)-ARCH(1). In all models that were used, the heteroskedasticity effect was eliminated.

Table 1: Traditional Unit Root Tests Results for the Return Series

	ADF (None)	PP (None)	KPSS (None)
R (Return)	-19.348 (0.000)	-19.320 (0.000)	0.198
1% Critique Measure	-3.449	-2.573	0.739
5% Critique Measure	-2.869	-1.942	0.463
10% Critique Measure	-2.571	-1.616	0.347
Result	Series is stationary	Series is stationary	Series is stationary

Source: Created by the authors in the Eviews program.

Table 2: Lee-Strazicich Unit Root Test for the Return Series

Lee-Strazicich	LS (Model Crash-Constant)	LS (Model Break-Constant and Trend)
μ (Constant)	-0.7788 [t: -3.0672]	-0.3691[t: -9.0734]
α (S{1}	-4.5283[t: -8.9205]	-3.449 [t: -10.0285]
k*	6	3
1% Critical Value	-4.073	-6.750
5% Critical Value	-3.563	-6.108
10% Critical Value	-3.296	-5.779
Result	Series is stationary under structural breaks.	Series is stationary under structural breaks.
Break dates (Time break):	2008:07; 2020:10	2001:06; 2020:08
	($\hat{\lambda}_1$=0.40, $\hat{\lambda}_2$=0.84)	($\hat{\lambda}_1$=0.16, $\hat{\lambda}_2$=0.83)

Note: Values in [ ] are the t-statistics values for the coefficients. k* represents the appropriate number of lags. The basic hypothesis is that the series is non-stationary under structural breaks. The lambda values in parentheses indicate the ratio of the observation value to the total number of observations at the break date.

Source: Created by the authors in the Winrats program.

Table 3: Carrion-i-Silvestre et al. (2009) Unit Root Test for the Return Series

	Carrion-i-Silvestre (2009)				
	PT	MPT	MZα	MSB	MZT
R	5.772	3.792	-96.672	0.093	-7.552
	(9.036)	(8.844)	(-46.773)	(0.117)	(-4.317)
Result:	Series is stationary under structural breaks.				
Break dates (Time break):	35. (1999:11), 58. (2001:10), 142. (2008:10), 285. (2020:09), 317. (2023:05).				

Note: The values in parentheses are critical values. Here, the PT, MPT, MZα, MSB, and MZT values are 5 different tests that are applied in this analysis. In the Carrion-i-Silvestre test, while the null hypothesis states that the series is stationary under structural breaks, the alternative hypothesis states that it is non-stationary under structural breaks. This situation is similar in the KPSS test. Additionally, as opposed to conventional hypothesis tests, in the case that the test statistic is greater than the critical value in the Carrion-i-Silvestre test, the null hypothesis cannot be rejected.

Source: Created by the authors in the Gauss program.

Table 4: ARMA Predictions for the Return Series

	ARMA(2,3)	ARMA(3,2)	ARMA(3,3)
c	0.007***	0.007***	0.005***
	(0.002)	(0.003)	(0.000)
ϕ_1	-0.555***	-0.565***	0.403***
	(0.000)	(0.000)	(0.000)
ϕ_2	-0.936***	-0.942***	-0.400***
	(0.000)	(0.000)	(0.000)
ϕ_3	-	-0.009	0.898***
		(0.899)	(0.000)
θ_1	0.558***	0.568***	-0.425***
	(0.000)	(0.000)	(0.000)
θ_2	0.982***	0.988***	0.421***
	(0.000)	(0.000)	(0.000)
θ_3	-0.009***	-	-0.983***
	(0.000)		(0.000)
Adj. R^2	0.061	0.071	0.092
F(Prob)	5.354*** (0.000)	6.015***(0.000)	6.752*** (0.000)
AIC	-3.316	-3.305	-3.331
SIC	-3.248	-3.204	-3.252
HQ	-3.288	-3.268	-3.300
JB	0.657(0.445)	0.661(0.442)	0.877(0.393)
BG LM	4.015 (0.134)	4.273 (0128)	0.841(0.656)
ARCH LM	6.700 (0.009)	6.703 (0.009)	6.106 (0.013)

Note: The values in parentheses are probability values for the coefficients. ***, **, and * show significance on the levels of 1%, 5%, and 10%, respectively.

Source: Created by the authors in the Eviews program.

Table 5: Forecasting Results of the ARMA-ARCH mixed type for the Return Series

Variables	ARMA(3,3)-ARCH(1)	ARMA(3,3)-EGARCH(1,1)	ARMA(3,3)-TGARCH(1,1)	ARMA(3,3)-AVGARCH(1,1)	ARMA(3,3)-APARCH(1,0)
c	0.006***	0.005***	0.006***	0.006***	0.005***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ϕ_1	0.269***	0.406***	0.242***	-0.556***	0.995***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ϕ_2	-0.305***	-0.401***	-0.289***	-0.238***	0.672***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ϕ_3	0.851***	0.900***	0.839***	0.482***	-0.336***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
θ_1	-0.285***	-0.427***	-0.253***	0.845***	-0.928**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.033)
θ_2	0.351***	0.423***	0.366***	0.157***	0.184***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
θ_3	-0.948***	-0.984***	-0.924***	-0.638***	0.693***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α_0	0.001***	0.027***	0.000	0.002	0.000
	(0.000)	(0.000)	(0.190)	(0.151)	(0.338)
α_1	0.171***	-	0.033***	0.336***	0.166
	(0.005)		(0.000)	(0.000)	(0.138)
β_1	-	-8.245**	0.109**	0.914***	-
		(0.011)	(0.040)	(0.000)	
γ_1	-	-	0.860	-	1.668***
			(0.227)		(0.000)
δ_1	-	0.082*	-	-	-
		(0.061)			
δ_2	-	-0.326	-	-	-
		(0.532)			
η_{11}	-		-	0.197**	-0.107
				(0.034)	(0.445)
η_{21}	-	-	-	-0.554***	-
				(0.000)	
T-DIST-DOF	19.999**	-	6.957***	-	6.443
	(0.024)		(0.000)		(0.227)
Adj.R^2	0.034	0.079	0.028	0.095	0.072
AIC	-3.413	-3.320	-3.591	-3.385	-3.171
SIC	-3.299	-3.206	-3.466	-3.524	-3.109
ARCH LM	0.558 (0.455)	1.445 (0.249)	0.083 (0.774)	0.315 (0.661)	0.619(0.379)
JB	0.277 (0.723)	0.186(0.844)	0.442(0.569)	0.335(0.673)	0.439(0.572)
Sign Bias	0.949(0.350)	0.948(0.343)	0.477(0.663)	0.604(0.536)	0.508(0.584)

Note: The values in parentheses are probability values for the coefficients. ***, **, and * show significance on the levels of 1%, 5%, and 10%, respectively.

Source: Created by the authors in the Gauss program.

However, Hamilton and Susmel (1994) stated that the ARCH and GARCH model structures overestimate volatility in conditional volatility cases, and thus, their forecasting performance is not adequate. According to Lamoureux and Lastrapes (1990), on the other hand, using a model structure that does not allow regime changes in parameter prediction for volatility may affect the reliability of the parameters negatively. Therefore, a specification that allows the structural change of ARCH parameters is needed. For this purpose, the Markov regime-switching-based ARCH model structures shown in Table-6 were separately predicted using R Project.

The parameters α and β in Table-6 are parameters that are included in the variance model. The sum of these Markov regime-switching-based parameters was smaller than the sums of the parameters in the standard ARCH-GARCH models (Results for each MS-GARCH model, respectively: Regime I: 0.859, 0.714, 0.198; Regime II: 0.999, 0.778, 0.504). These results showed that the MS-GARCH family of models did not forecast high persistence in low-volatility periods (Regime I), as expected. In the MS-GARCH structure created with two regimes, the initial system was described as the deep-fluctuation regime, and the following system was expressed as the huge-fluctuation regime. Among the 3 models that were used in forecasting, according to the importance degrees with the parameters and the information criterion, the optimum model was determined as the MS-EGARCH(1,1)-GED model. The significant δ parameter in the model in

Table 6: Forecasting Results of the MRSM-ARCH mixed type for the Return Series

	MS-GARCH(1,1)-std		MS-EGARCH(1,1)-ged		MS-GJRGARCH(2,1)-std	
	Regime I	Regime II	Regime I	Regime II	Regime I	Regime II
α_0	0.000***	0.000***	-0.075***	-0.006***	0.000***	0.001
	(0.007)	(0.227)	(0.000)	(0.008)	(0.000)	(0.365)
α_1	0.032*	0.005	-	-	0.005	0.010
	(0.073)	(0.269)			(0.263)	(0.309)
β_1	0.827***	0.994***	0.683***	0.691***	0.007	0.503
	(0.000)	(0.000)	(0.000)	(0.000)	(0.230)	(0.256)
γ_1	-	-	-	-	0.186*	0.000
					(0.094)	(0.489)
δ_1	-	-	0.106***	0.093**	-	-
			(0.000)	(0.017)		
$GED-par.$	-	-	6.104***	13.449***	-	-
			(0.000)	(0.000)		
$STD-par.$	3.785**	14.891***	-	-	58.872***	10.484***
	(0.036)	(0.000)			(0.000)	(0.000)
	Transition matrix		Transition matrix		Transition matrix	
	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2
t|k=1	0.9872	0.0128	0.9003	0.0997	0.9644	0.0356
t|k=2	0.0275	0.9725	0.0148	0.9852	0.0459	0.9541
	Stable probabilities		Stable probabilities		Stable probabilities	
	State 1	State 2	State 1	State 2	State 1	State 2
	0.7049	0.2951	0.8759	0.1241	0.6862	0.3138
LL	509.443		511.849		501.489	
AIC	-982.716		-993.488		-972.881	
BIC	-952.875		-946.657		-926.350	
BG-LM	2.114(0.396)		1.083(0.504)		1.949(0.427)	
ARCH-LM	0.889 (0.274)		0.661(0.499)		0.819(0.302)	
JB	1.027 (0.288)		0.774(0.402)		0.947(0.305)	
Sign Bias	0.683(0.877)		0.913(0.643)		0.725(0.706)	

Note: The values in parentheses are probability values for the coefficients. ***, **, and * show significance on the levels of 1%, 5%, and 10%, respectively.
Source: Created by the authors in the R Studio program.

question showed the existence of a pry or asymmetry impact in the model.

In the comparisons of the δ_1 values of Regime I and Regime II [Regime I δ_1 value (0.106) is greater than Regime II δ_1 value (0.093)], it was concluded that index returns showed abrupt and high-level reactions to the shocks at the market in the deep-venture regime (Regime I), but they became stable following the shocks that were experienced in the huge-venture regime (Regime II). The opposite case was valid for the δ_2 values. The same results could also be derived from the δ_1 coefficients that indicate the persistence of volatility (Regimes I and II). This situation was applicable to both regimes. The approach of δ_1 towards 1 increases the persistence of volatility. This result showed that abrupt increases in returns, as well as shocks that also increase volatility, would disappear in the short and long terms, and the returns would reach a balance.

In the transition matrix showing the Markov forecasting results, "k=1" and "k=2" refer to Regime I and Regime II, respectively. In the selected MS-EGARCH(1,1)-GED model, the transition probability referring to staying in the regime with deep venture (Regime I) once the series was already in Regime 1 was found as 90.03%. Furthermore, while the series was already in Regime II, the transition probability—which refers to remaining in the high-risk regime (Regime II)—was 98.52%. It was found that there was an 0.997% chance of moving from the high-risk regime to the low-risk regime and a 0.148% chance of moving from the deep-venture regime to the huge-venture regime. This finding would imply that there will be strong transitions between the opposing regimes and frequent fluctuations in volatility over brief periods of time.

Finally, considering the unconditional probability values, these values that are known as stable probabilities refer to the probability of the limit values of both regimes at infinity. In other words, in the long term, the unconditional probability values indicate the level of stability that will not allow transition to a previous regime again. For the selected MS-EGARCH(1,1) model in Table-6, while the unconditional probability coefficient for Regime I was 0.8759, it was calculated as 0.1241 for Regime II. The higher stability of

Regime I compared to Regime II suggested that Regime II had a trend towards Regime I. The high value in State 1 indicated that the sudden fluctuations in BIST 100 returns would reach a balance in the long term. If the value in State 2 were high, it would be interpreted as that the fluctuations in the returns would continue in the long period.

To identify the time periods during which the deep- and huge-fluctuation regimes were observed, the Iterated Cumulative Sums of Squares (ICSS) technique was utilized (Inclan and Tiao, 1994:918). This technique is used to identify breaks that will occur as a result of abrupt shocks that can be seen in the variance of a time series. On that reason, the ICSS technique was applied to the BIST 100 Index return series using the RATS software, and the result shown in Figure-2 was obtained.

The 1st break was in December 2001, and the 2nd break occurred in March 2010.

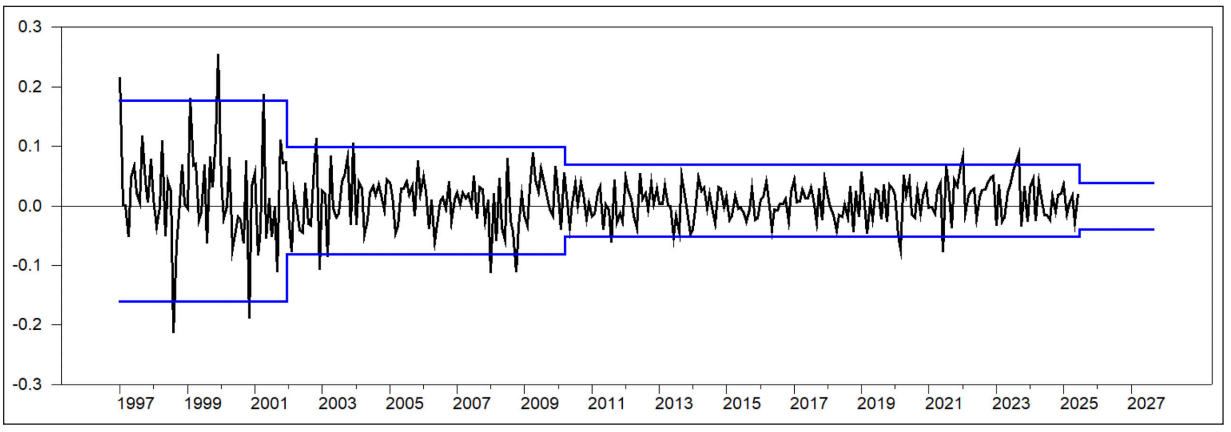

Figure 2: Dates of Variance Breaks Captured in the Volatility of the Return Series.

Source: Created by the authors in the Winrats program.

The reasons for the 1st break which occurred in December 2001 may be the economic crisis experienced in Turkey in 2001 subsequent the world financial downturn in 2000 and the uncertainties faced before the general elections in November 2002. The reasons for the 2nd break which occurred in March 2010 may be the negative growth experienced in Turkey in the second half of 2009, the decrease in inflation as a result of the 3-fold increase in the budget balance in the negative direction in contrast to the previous year, as well as the increase in per capita income and decrease in foreign currency values in the first months of 2010.

The return (R) series (variable) was forecasted in the MATLAB program using ANNs by trying different layers. Using the error term of the ARMA(3,3) model that was the optimum model for the return series as input and the return series itself as output, ANN modeling was performed. Due to the large number of data points (n=342), using the cross-validation method, 96% of the data points were dispersed in the training set, and 4% were separated in the test set.

At the model prediction stage, the rectified linear unit transfer function was chosen for the buried layer's neurons, a linear transfer function and the backpropagation algorithm were selected for the neurons in the output layer, and the results shown in Table-7 were obtained.

In the selection of the optimum network structure with the prediction models, the network with the lowest Mean Squared Error (MSE) value was used. MSE is calculated using the formula in Equation 21:

$$\text{MSE:} \quad \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \tag{21}$$

According to the prediction results of the ANNs with different architectures, the optimum network architecture with the lowest MSE values was determined as the Cascade-Feedforward ANNs. The MSE values of this selected network is marked with *.

In this study, in addition to these predictions for the return series, predictions were also made using hybrid modeling techniques in the MATLAB program. For this, the values in the variance equation of the ARCH model were multiplied by the error terms derived from the model, and the results were used as input for the ANN. The same method was also used for the MS-EGARCH model. The results are shown in Table-8:

In the hybrid model prediction steps, while the MSE values of the three models were close to each other, the MS-EGARCH-ANN (Regime II) hybrid model can be considered the optimum model.

Table 7: ANN Forecasting Results for the Return Series

Network Type	ANN Architecture	Training Algorithm	MSE
Cascade-Feddforward	1-13-1 (Single-Layer)	Levenberg-Marquardt	0.00026*
Elman Backpropagation	1-22-1 (Single-Layer)	Bayesian Regularization	0.00048
Feedforward	1-15-1 (Single-Layer)	Levenberg-Marquardt	0.00031
Recurrent	1-14-1 (Single-Layer)	Levenberg-Marquardt	0.00040
Recurrent	1-9-5-1 (Double-Layer)	Polak-Ribiére Conjugate Gradient	0.00044
Recurrent Backpropagation NARX	1-8-1(Single-Layer)	Powell-Beale Conjugate Gradient	0.00051
Radial Basis	1-9-4-1(Double-Layer)	Levenberg-Marquardt	0.00029

Note: ANN Architecture numbers indicate the number of neurons at the input and output levels, respectively. Values in parentheses indicate the number of layers. * indicates the optimal MSE value for the developed network structure.
Source: Created by the authors in the Matlab program.

Table 8: Forecasting Results of the Hybrid Models for the Return Series

Network Type	ANN Architecture	Training Algorithm	MSE
ARCH-ANN	1-16-1 (Single-Layer)	Levenberg-Marquardt	0.00016*
MS-EGARCH-ANN (Regime I)	4-11-7-1 (Double-Layer)	Levenberg-Marquardt	0.00015*
MS-EGARCH-ANN (Regime II)	4-10-5-1 (Double-Layer)	Levenberg-Marquardt	0.00011*

Note: ANN Architecture numbers indicate the number of neurons at the input and output levels for hybrid models, respectively. Values in the parentheses indicate the number of layers for hybrid models. * indicates the optimal MSE values for the developed hybrid models.
Source: Created by the authors in the Matlab program.

Table 9: Forecasting Performance Comparison for the Return Series

Model No	Return (R)	RMSE	MAPE	Theil-U
1	ARCH	0.027	78.391	0.744
2	MS-EGARCH	0.016	39.964	0.221
3	ANN (Cascade-Forward)	0.008	17.392	0.067
4	**ARCH-ANN**	**0.005**	**8.462**	**0.049**
5	**MS-EGARCH-ANN (Regime I)**	**0.005**	**12.506**	**0.047**
6	**MS-EGARCH-ANN (Regime II)**	**0.004**	**10.451**	**0.045**

Note: The dark colored models are the structures with the best forecasting performance for the return series.
Source: Created by the authors in the Matlab program.

In fact, the accuracy and consistency of forecasting performance can be assessed using a set of performance criteria. These criteria including RMSE (Root Mean Square Error), MAPE (Mean Absolute Percentage Error), and U (Theil's Inequality Coefficient) are calculated using the formulae given in Equations 22, 23, and 24:

$$\text{RMSE: } \sqrt{\frac{\sum_{i=1}^{n}(Y_i-\hat{Y}_i)^2}{n}} \tag{22}$$

$$\text{MAPE: } \frac{1}{n}\sum_{i=1}^{n}\left|\frac{Y_i-\hat{Y}_i}{Y_i}\right| x100 \tag{23}$$

$$\text{U: } \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}[(Y_i-\hat{Y}_i)]}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}Y_i^2}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\hat{Y}_i^2}} \tag{24}$$

The forecasting performance values of all models that were predicted in the study were calculated in Excel using these criteria, and the results are presented in Table-9.

According to the results of the comparisons made based on the performance criteria, the most reliable forecasting performance for the return variable was seen in the ARCH-ANN hybrid model numbered 4 and the MS-EGARCH-ANN hybrid models numbered 5 and 6 (Regimes I and II).

Finally, the actual values of the return variable and the forecasting values obtained with the hybrid model predictions were calculated in Excel and are shown in Table-10.

Table-10 shows that the hybrid models produced the closest values to the actual ones in forecasting, and they had high forecasting performance. It should be noted that the geometric mean of the absolute values of these three

Table 10: Actual and Hybrid-Forecasted Values of the Return Variable

Months	Actual	Hybrid (ARCH-ANN) Forecast (4)	Hybrid (MS-EGARCH-ANN Regime I) Forecast (5)	Hybrid (MS-EGARCH-ANN Regime II) Forecast (6)	Geometric Mean (4,5,6)
January 2025	0.0354	0.0291	0.0325	0.0350	0.0321
February 2025	-0.0146	-0.0190	-0.0166	-0.0143	-0.0165
March 2025	0.0028	0.0037	0.0032	0.0026	0.0031
April 2025	0.0167	0.0157	0.0163	0.0164	0.0161
May 2025	-0.0278	-0.0261	-0.0241	-0.0211	-0.0236
June 2025	0.0197	0.0149	0.0178	0.0191	0.0171
Total Abs. Dev.	-	0.0191	0.0113	0.0085	0.0129

Note: In this table, actual returns are compared with appropriate hybrid model techniques. These values are also statistically compared with the geometric mean of the hybrid model.
Source: Created by authors in the Matlab program

forecasts was calculated to obtain a single forecasting performance result instead of three results, and by substituting the sign of the forecast in, the forecasts in the last column were obtained. Accordingly, for example, hybrid model number 6 showed almost point estimation feature in all months except May. Considering that the deviations in the other months were also very small, it may be stated that the predictions made based on the hybrid models had high performance levels.

This finding shows that when financial time series are sensitive to regime changes and are backed by artificial neural networks' capacity to recognize nonlinear patterns, forecasting performance rises dramatically. (Marcucci, 2005; Bildirici and Ersin, 2014; Augustyniak, 2014; Kula and Baykut, 2017; Kutlu and Karakaya, 2019; Tan et al., 2021; He et al., 2022; Kontopolou et al., 2023; Cappello et al., 2025; Agarwal et al., 2025) Similar models in the literature are typically restricted to the use of single or binary structures (e.g., ARIMA, GARCH, ANN, ARIMA-ANN, MS-GARCH). Nevertheless, compared to previous research, this study's modeling of regime transitions using a probability-based structure and ANN-based learning of the impact of these changes on volatility shows that a more dynamic and layered structure functions effectively.

Additionally, investor confidence, market stability, and capital flows in the Turkish economy are all significantly impacted by the model's excellent success in predicting the BIST100 index. Accurate modeling of regime volatility offers important insights into the examination of economic cycles as the index represents not only the worth of listed businesses but also overall economic mood and expectations. Divergences between regimes 1 and 2 in particular show how market responses change depending on the economic environment, making it possible to simulate elements like investor behavior, capital inflows and outflows, and risk appetite during times of economic boom or recession. As a result, extremely precise BIST100 projections may be used as a first step in determining consumer confidence, real sector expectations, and the overall trajectory of financial markets.

CONCLUSION and RECOMMENDATIONS

There are many dangers associated with financial markets, thus reducing ambiguity is crucial. In this case, volatility is a crucial signal. Because ARCH models may capture asymmetric market shocks, they are frequently employed to evaluate volatility. However, ARCH has been integrated with Markov regime-switching models due to shortcomings such as overestimation of volatility and poor forecasting.

The search for alternative methods that can model volatility better in recent years has resulted in the concept of hybrid models (techniques) that combines the advantages of two different model structures. The success of hybrid models in forecasting volatility and the limited number of studies in this field gave rise to the need for conducting this study.

Using monthly BIST 100 return data, this study examines the performance of hybrid models that combine ARCH and ANN as well as Markov regime-switching ARCH models with ANN. Heteroskedasticity was found in the initial ARMA model estimates, which made ARCH-based methods necessary.

In the next step, alternative ARCH models were predicted, and it was decided that the ARCH model was the optimum model. Afterward, among the Markov regime-switching-based ARCH models that were run, the MS-EGARCH(1,1)-GED model was determined to be the optimum model. To achieve better forecasting

performance, hybrid models which included the combination of the ARCH model and ANNs and the combination of the MS-EGARCH(1,1) model and ANNS were predicted.

Using a single-input, single-output structure, models were trained with various network types. Cascade-Feedforward ANN yielded the lowest mean squared error among the tested architectures.

Consequently, the forecasting performance values of 6 different models, including 1) ARCH, 2) MS-EGARCH, 3) ANN (Cascade-Feedforward), 4) ARCH-ANN, 5) MS-EGARCH-ANN (Regime I), and 6) MS-EGARCH-ANN (Regime II), were compared. Based on RMSE, MAPE, and Theil's U, the best results were obtained from hybrid models 4, 5, and 6. A single forecast was derived using the geometric mean of their absolute forecast values. As the 6-month forecasts closely matched actual values, the models were found to be highly effective.

The MSE-GARCH-ANN hybrid model achieved high accuracy in forecasting BIST100 volatility, effectively capturing regime shifts. This enables early identification of market uncertainties, helping institutional investors optimize portfolios and individuals select hedging strategies more consciously. Regime-based volatility models can help policymakers time monetary and fiscal actions more effectively. Early signals from indices like BIST100 enable timely interventions to preserve macroeconomic stability. Thus, this model can serve as an early warning tool for central banks and regulators, highlighting the importance of incorporating AI-driven analysis into financial policy design. In conclusion, the developed hybrid approach not only contributes to technical success but also provides tangible benefits in terms of market stability, investment security, and policy effectiveness. Therefore, the model can be considered a reference method both in academic literature and in applied economic and political decision-making processes.

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APPENDİX

Table A.1. Literature Review Summary

Author/Authors	Dataset and Method	Similiarity	Difference
Cai (1994)	1972:2-1984:8 USA Treasury Bond: MS-ARCH	Regime-switching mechanism Structural changes in volatility dynamics	Core model components Methodological depth and flexibility Application and data diversity Estimation methods
Hamilton and Susmel (1994)	1987:10-1993:12 USA with weekly stock returns: MS-ARCH	Sensitivity to regime change Volatility prediction Alternative model comparison	Modeling depth and AI integration Application and data diversity How to use model outputs
Gray (1996)	1970:1-1994:4 USA Treasury Bond: MS-GARCH	Regime-transition volatility modeling Critical approach to traditional models	Width of model structure and level of hybridization Combining predictive performance and model output
Marcucci (2005)	1995:1-2004:8 USA with weekly stock returns- GARCH type models and MS-GARCH	Regime-transition volatility modeling Time Series Data and Financial Market Focus	Artificial Intelligence Integration Combination of Model Outputs Economic and Political Interpretation
Hu and Shin (2008)	1999.12-2007.03 weekly stock market index data of developing countries in East Asia- MSGARCH	Focus on Volatility Forecasting Combined Use of Artificial Intelligence and Econometric Methods Measuring Model Performance	Structure of the Model (Level of Hybridity) Regime Transitions and Asymmetric Volatility
Bildirici and Ersin (2014)	2000.1-2013.4 the daily stock returns in an emerging market, the Istanbul Stock Index- MSARMA-GARCH type models and MS-ARMA-FI-APGARCH-RNN	Focus on Volatility Forecasting Hybridization with Artificial Neural Networks (ANN) Focus on the Turkish Market Similarity in Results	Technical Depth and Structure of the Model Scope of Regime Modeling Modeling Purpose and Application Orientation
Augustyniak (2014)	MS-GARCH model: S&P 500 price index weekly October 28, 1987 to October 31, 2012, and S&P 500 price index daily May 20, 1999 to April 25, 2011.	Using the Regime-Switching GARCH Model Forecast Performance Analysis	Artificial Intelligence Layer (ANN) Modeling Depth and Hybridity Level Economic Application and Implications
Abounoori et al. (2016)	1999.1-2015.2 Tehran Stock Exchange – MSGARCH type	Using the Regime-Switching Model Forecast Performance Comparison	Artificial Intelligence and Hybridization Level Model Fit and Distribution Type Interpretation and Economic Inference Dimension
Kula and Baykut (2017)	August 31, 2007-December 31, 2015 Borsa Istanbul Corporate Governance Index (XKURY) and the Fear Index (Chicago Board Options Exchange Volatility Index-VIX-ARDL Model	Regime-Aware Modeling Application on Turkish Markets	Using Artificial Intelligence Depth and Hybrid Structure in Modeling Economic and Political Implications
Çavdar and Aydın (2017)	03.03.2014- 10.03.2017- Borsa Istanbul Corporate Governance Index (XKURY)- ARCH, GARCH, SWARCH	Borsa Istanbul Data Set Volatility Analysis Use of the Regime Transition Model	Model Dept and Artificial Intelligence Forecast performance and economic interpretation

Korkpoe and Howard (2019)	January 4, 2011 - 29 December 2017- Botswana, Ghana, Kenya and Nigeria-BMS-EGARCH, BMS-GJR-GARCH	Regime Transition Volatility Modeling	Artificial Intelligence Component Forecast Synergy Economic Application Layer Data Definition and Frequency
Kutlu and Karakaya (2019)	In three periods from 05/02/2003 to 09/14/2018- Borsa Istanbul Tourism Index, MS-ARCH Model	Volatility modeling sensitive to regime changes Emerging market application	Model Components and Depth Use of Artificial Intelligence Forecast Combination Strategy Performance Analysis and Error Measures
Kaya and Yarbaşı (2021)	BIST100 index closing data for the period of 03.01.1988-20.04.2018- MSGARCH Model	Using MS-GARCH and Regime Analysis Emerging Market Focus Empirical Assessment of Model Performance	Model Structural Depth and Artificial Intelligence Component Combination of forecasts and performance metrics
Tan et al. (2021)	August 1, 2010 - July 31, 2018, BTC price, TV-MS GARCH Model	Volatility Approach Modeling Regime Shifts Implementation with Emerging Market Data	Regime transition probability structure Use of exogenous variables Model combination and forecasting strategy
He et al. (2023)	7 April 2008 - 21 September 2020, EU ETS- Shanghai composite index-BTC price, ARMA-CNN-LSTM	Model Combination Approach Model Linear and Nonlinear Data Together	ANN Architectural Diversity and Regime Methodology Model Performance Measures Economic application
Kontopoluo et al. (2023)	Financial, healthcare, weather, utilites, network traffic data- hybrid techniques ARIMA, SVM, RNN and LSTM	Comparison of Statistical and Artificial Intelligence Approaches Focus on Model Performance & Comprehensive Comparisons	Regime, Data and Market Focus Model Performance Measures Structuring the prediction Policy Implications
Cappello et al. (2025)	2 January 2019- 26 September 2023, Unicredit SpA stock, the Bitcoin prices and the nominal EUR/USD exchange rates, ARIMA, ANN, ARIMA-LSTM, ARIMA-GRU	Comparison of Statistical and Artificial Intelligence Methods Increasing Prediction Accuracy with Hybrid Approaches Prediction Performance Evaluation	Model Synergy and Ensembling with Regime Transition Modeling Data Focus and Application Area Economic/Political Recommendations
Agarwal et al. (2025)	December 13, 2007- December 12, 2017, daily closing prices of the Hong Kong Hang Seng Index (HSI), Standard and Poor's 500 (S&P500) Index of the US, Bombay stock exchange (SENSEX) of India, and North America West Texas Intermediate (WTI), Machine and Deep Learning Hybrid	Data Parsing + AI-Based Hybrid Approach Comparison of Hybrid Model Performance	ANN Architecture Structuring the prediction Policy Implications Model Performance Evaluation

Table A.2. List of Abbreviations

Abbrevations	Explanation
ARMA	Autoregressive Moving Average
ARIMA	Autoregressive Integrated Moving Average
ARCH	Autoregressive Conditional Heteroskedasticity
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
AVGARCH	Absolute Value Generalized Autoregressive Conditional Heteroskedasticity
TGARCH	Threshold Generalized Autoregressive Conditional Heteroskedasticity
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedasticity
APARCH	Asymmetric Power Autoregressive Conditional Heteroskedasticity
GJR-GARCH	Glosten-Jagannathan- Runkle Generalized Autoregressive Conditional Heteroskedasticity
SWARCH	Swtiching Autoregressive Conditional Heteroskedasticity
MS-GARCH	Markow Regime Switching Generalized Autoregressive Conditional Heteroskedasticity
ANN	Artificial Neural Network
TV-MS-GARCH	Time Varying Markow Regime Switching Generalized Autoregressive Conditional Heteroskedasticity
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MSE	Mean Squared Error
RMSE	Root Mean Squared Error
GED	Generalized Error Distribution
CNN	Convolutional Neural Network
LSTM	Long Short Term- Memory
GRU	Gated Recurrent Unit
SVM	Support Vector Machine
ADF	Augmented Dickey Fuller
PP	Philips Peron
KPSS	Kwiatkowski–Phillips–Schmidt–Shin
STD	Student t distribution
LS	Lee- Strazicich
ICSS	Iterated Cumulative Sums of Squares